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Inflation from a nonlinear magnetic monopole field nonminimally coupled to curvature

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Abstract. In the context of nonminimally coupled $f(R)$ gravity theories, we study early inflation driven by a nonlinear monopole magnetic field which is nonminimally coupled to curvature. In order to isolate the effects of the nonminimal coupling between matter and curvature we assume the pure gravitational sector to have the Einstein-Hilbert form. Thus, we study the most simple model with a nonminimal coupling function which is linear in the Ricci scalar. From an effective fluid description, we show the existence of an early exponential expansion regime of the Universe, followed by a transition to a radiation-dominated era. In particular, by applying the most recent results of the Planck collaboration we set the limits on the parameter of the nonminimal coupling, and the quotient of the nonminimal coupling and the nonlinear monopole magnetic scales. We found that these parameters must take large values in order to satisfy the observational constraints. Furthermore, by obtaining the relation for the graviton mass, we show the consistency of our results with the recent gravitational wave data GW170817 of LIGO and Virgo.

Contents

1 Introduction

Cosmic inflation is the currently accepted paradigm which provides an elegant solution to the well-know problems of standard Big Bang cosmology [1–3]. Furthermore, it allows us to explain the origin of primordial fluctuations, which are the seeds for all observed structures in the Universe [4, 5]. The early inflationary phase can be studied in the context of scalar field models by maintaining General Relativity (GR) as the gravitational theory, where a canonical scalar field ϕ (inflaton) drives a quasi-exponential expansion through an arbitrary flat potential $V(\phi)$ [6, 7]. Another approach is obtained in the context of modified gravity models (e.g. $f(R)$ gravity theories [8–15]), where higher-order curvatures terms, motivated from quantum corrections, are also added to the Einstein-Hilbert action of GR.

The most general scenario, which mixes the best attributes and potentialities of each one of the above approaches, is obtained by allowing a nonminimal coupling between curvature and matter [16–21]. In these nonminimal matter-curvature coupling theories, besides of an arbitrary function $f_1(R)$ in the gravity sector, one also introduces a nonminimal coupling by adding to the action a term of the form $f_2(R)\mathcal{L}_m$, where $f_2(R)$ is an arbitrary function of the scalar curvature and $\mathcal{L}_m$ is the matter Lagrangian density. It has been shown in ref. [22] that these theories allow to mimic known dark matter density profiles through an appropriate power-law coupling term with negative power index, in such a way that it is guaranteed the dominance of dark matter, before the dark energy-dominated era. A study of the dynamics of cosmological perturbations in these theories has been performed in ref. [23], where the authors have examined how the evolution of cosmological perturbations is affected due to the presence of a nonminimal coupling between curvature and matter on a homogeneous and isotropic Universe, and hence the formation of large-scale structure. The late-time acceleration of the Universe was studied in refs. [24, 25], where it has been shown that the current cosmic acceleration may have originated from a nonminimal curvature-matter coupling, and, through mimicking, it can provide a viable scenario for the unification of dark energy and dark matter. Moreover, recently in ref. [26], the authors have investigated an inflationary scenario in this context, by taking the matter Lagrangian density to be that one of a canonical scalar field. Thus, they have shown that the spectrum of primordial perturbations and the inflationary dynamics can be significantly modified once that the energy density of

matter exceeds a critical value which is determined by the scale of the nonminimal coupling term. Finally, the detection of gravitational waves with an electromagnetic counterpart (GW170817) [27], emitted by a binary neutron star merger, has put a strong constraint on the speed of tensor polarization modes, which becomes extremely very close to the speed of light with a deviation smaller than 10^{-15} [28–34]. So, in ref. [35], it was also recently analysed the propagation of gravitational waves modes in nonminimal curvature-matter theories, for the cases of cosmological constant and dark energy-like fluid as matter source, showing a good agreement with the recent gravitational wave constraints.

Nevertheless, in choosing the matter Lagrangian for the very early Universe, the scalar field Lagrangian is not the only possibility. Big Bang, which was the starting point of the Universe, has a singularity [36], and this singularity breaks all the physics laws [37], therefore being this the big gap in the understanding of nature of the Universe [6]. Nowadays, the various studies on nonlinear electromagnetic fields are done to remove the singularity of the Universe [38–40]. The main idea is to modify the Maxwell equations smartly to remove singularity [41]. Using this fundamental method at strong fields such as early universe, nonsingular universe models have been found, namely magnetic universe [42], however sometimes it is difficult to preserve the conformal invariance of Maxwell fields after the modification of the electromagnetic fields. Magnetic fields are believed to have played a large part in the dynamics of the evolution of our universe. However, little is known about the existence of magnetic fields when the universe was very young. Observations have manifested the existence of magnetic field in the universe, ranging from the stellar scale 10^{-5} pc to the cosmological scale 10^4 Mpc [43, 44]. In particular, the magnetic field on large scales (≤ 1 Mpc) is deemed to be produced in the early universe [45], namely, the primordial magnetic field. By the recent CMB observations, the strength of the magnetic field is smaller than a few nano-gauss at the 1 Mpc scale [46]. Additionally, the $\gamma - ray$ detections of the distant blazars imply that the magnetic field should be larger than 10^{-16} gauss on the scales $1 - 10^4$ Mpc [47].

Although there are no direct observations of primordial magnetic fields, it is believed that they existed because may have been needed to seed the large magnetic fields observed today. Theories also disagree on the amplitude of primordial magnetic fields. There are currently several dozen of theories about the origin of cosmic magnetic fields for instance primordial vorticity plasma (vortical motion during the radiation era of the early Universe, vortical thermal background by macroscopic parity-violating currents), quantum-chromo-dynamics phase transition, first-order electroweak phase transition via a dynamo mechanism, etc ([48] and references therein). In the last scenario, seed fields are provided by random magnetic field fluctuations which are always present on a scale of the order of a thermal wavelength. It refers to stochastic background with a nonzero value of $< B^2 >$. Stochastic magnetic fields are the cosmic background with the wavelength smaller than the curvature. Thermal fluctuations in a dissipative plasma, i.e. plasma fluctuations, could source stochastic magnetic fields on a scale larger than the thermal wavelength [48]. Thus, there are the stochastic fluctuations of the electromagnetic field in a relativistic electron-positron plasma. Although homogeneous magnetic fields can affect the isotropy of the Universe, i.e. the energy-momentum tensor can become anisotropic which could cause an anisotropic expansion law and modify the CMB spectrum, the effect on the Universe geometry (isotropy) of magnetic fields tangled on scales much smaller than the Hubble radius are negligible [48]. Thus, averaging the magnetic fields, which are sources in general relativity [49], give the isotropy of the Friedman-Robertson-Walker (FRW) spacetime. Symmetry arguments also can be used to construct a triplet of mutually orthogonal (magnetic) vector fields which respects in an exact form the spatial ho-

mogeneity and isotropy of the Universe [50–52]. Although, N randomly oriented vector fields lead to a slightly anisotropic Universe with a degree of anisotropy of order $1/\sqrt{N}$ at the end of inflation [53], one can avoid a large anisotropy by considering a large number of randomly oriented vector fields in a sufficiently large time-dependent three-volume, as through of the average procedure used in refs. [41, 54]. So, as it has been shown in refs. [38–42, 55–66], a nonlinear magnetic monopole field can also play an important role in the inflationary dynamics, despite breaking the conformal invariance of the Maxwell theory [39, 40]. Furthermore, since a nonminimal coupling with curvature can affect significantly the physical results obtained in these models [26], here, we are going to study a nonlinear magnetic monopole field nonminimally coupled to curvature and its implications for the early inflationary phase.

The paper is organized as follows. In section II, we define our model on the nonminimal matter-curvature theories and in section III, we study the effect of nonlinear electrodynamics on the cosmology using the nonminimal matter-curvature coupling. In section IV, the evolution of the Universe is studied and we introduce an effective perfect fluid description, which allows us to put the modified Friedmann equations in its effective standard form, by identifying the relevant cosmological parameters. Thus, in order to study the dynamics of inflation, we introduce the slow-roll parameters, along with the spectral indices to comparison with observational data. Finally, in section V, we conclude our paper.

2 Nonminimal matter-curvature theories

Our starting point is the action principle [19]:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} f_1(R) + f_2(R)\mathcal{L}_m \right], \tag{2.1}$$

where $\kappa^2 = 8\pi G$, $f_1(R)$ and $f_2(R)$ are functions of the Ricci scalar, and $\mathcal{L}_m$ is the matter Lagrangian density.

Varying the action (2.1) with respect to the metric we obtain the field equations

$$(R_{\mu\nu} - \Delta_{\mu\nu}) \left[F_1 + 2\kappa^2 \mathcal{L}_m F_2 \right] - \frac{1}{2} f_1 g_{\mu\nu} = \kappa^2 f_2 T_{\mu\nu}^{(m)}, \tag{2.2}$$

where we have defined $F_i(R) = f_i'(R)$, being that prime represents the derivative with respect to the Ricci scalar, and $\Delta_{\mu\nu} = (\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box)$. The matter energy-momentum tensor is defined as

$$T_{\mu\nu}^m = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \tag{2.3}$$

From the field equations (2.2) and by using the Bianchi identities we can show that the matter energy-momentum tensor satisfies the conservation law

$$\nabla^\mu T_{\mu\nu}^m = \frac{F_2}{f_2} \left[g_{\mu\nu}\mathcal{L}_m - T_{\mu\nu}^m \right] \nabla^\mu R, \tag{2.4}$$

which describes an exchange of energy and momentum between matter and the higher derivative curvature terms [19]. In the next section we will assume that the matter Lagrangian $\mathcal{L}_m = \mathcal{L}_{NMM}$ is the Lagrangian of a nonlinear magnetic monopole field.

3 Non-linear magnetic monopole fields and cosmology

The Lagrangian density of the non-linear magnetic monopole is given by [60]

$$\mathcal{L}_m = \mathcal{L}_{NMM} = -\frac{\mathcal{F}}{1 + 2\beta\mathcal{F}}, \tag{3.1}$$

where $\mathcal{F}$ is the Maxwell invariant. Since the matter Lagrangian does not depend on the derivatives of the metric, and using the definition (2.3), the matter energy-momentum tensor is written as [58]

$$T_{\mu\nu} = -K_{\mu\lambda}F_\nu{}^\lambda + g_{\mu\nu}\mathcal{L}_{NMM}, \tag{3.2}$$

with

$$K_{\mu\lambda} = \frac{\partial\mathcal{L}_{NMM}}{\partial\mathcal{F}}F_{\mu\lambda}. \tag{3.3}$$

We suppose that there is a dominant stochastic magnetic fields on the cosmic background and their wavelengths are smaller than the curvature so that the averaging of EM fields are used as a source of Einstein equations coupled to non-minimal coupled to curvature [49]. The averaged electromagnetic fields are given as [41, 54]:

$$\langle E \rangle = \langle B \rangle = 0, \qquad\qquad \langle E_i B_j \rangle = 0, \tag{3.4}$$
$$\langle E_i E_j \rangle = \frac{1}{3}E^2 g_{ij}, \qquad\qquad \langle B_i B_j \rangle = \frac{1}{3}B^2 g_{ij},$$

which $\langle\ \rangle$ is the averaging brackets. We consider it to take average volume and this volume is larger value than the radiation wavelength, and also smaller than the curvature.

Thus, in the pure non-linear magnetic monopole case, such that $E^2 = 0$, from eq. (3.2) we obtain the energy density $\rho = -T^0{}_0$ and the pressure $p = T^i{}_i/3$ of the non-linear monopole magnetic field [56]

$$\rho = -\mathcal{L}_{NMM}, \tag{3.5}$$
$$p = \mathcal{L}_{NMM} - \frac{2B^2}{3}\frac{\partial\mathcal{L}_{NMM}}{\partial\mathcal{F}}, \tag{3.6}$$

where $\mathcal{L}_{NMM}$ is defined in eq. (3.1) with $\mathcal{F} = B^2/2$.

Then the energy density ρ and pressure p of the NMM is obtained as follows:

$$\rho = \frac{B^2}{2B^2\beta + 2}, \tag{3.7}$$
$$p = \frac{-3B^4\beta + B^2}{6(B^2\beta + 1)^2}. \tag{3.8}$$

Assuming that $f_1(R) = R$ and $f_2(R) = 1 + \lambda R$, the field equations (2.2) are simplified as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2\Big[-2\lambda\mathcal{L}_{NMM}R_{\mu\nu} + (1 + \lambda R)T_{\mu\nu} +$$
$$+ 2\lambda\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)\mathcal{L}_{NMM}\Big]. \tag{3.9}$$

We introduce the flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j, \tag{3.10}$$

where $a(t)$ is the scale factor. By using this metric (3.10) in the field equations (3.9), we obtain the following modified Friedmann equations

$$\frac{3H^2}{\kappa^2} = \frac{B\left[12\lambda H\dot{B} + B\left(1 + \beta B^2\right)\left(1 + 6\lambda H^2\right)\right]}{2(\beta B^2 + 1)^2}, \tag{3.11}$$

$$\dot{H} = \left[\frac{1}{3}B^2\left(6\beta\left(\kappa^2\lambda - \beta\right)\dot{B}^2 + 12\kappa^2\lambda H^2 + \kappa^2\right)\right.$$
$$\left. - \kappa^2\lambda B\left(\ddot{B} - H\dot{B}\right) - \left(\kappa^2\lambda + 2\beta\right)\dot{B}^2\right] \tag{3.12}$$
$$\times\left[\beta\left(\kappa^2\lambda - \beta\right)B^4 - \left(\kappa^2\lambda + 2\beta\right)B^2 - 1\right]^{-1} - \frac{2\beta\dot{B}^2}{\beta B^2 + 1}.$$

where the Hubble constant is defined as $H = \frac{\dot{a}}{a}$.

On other hand, from the energy conservation law in eq. (2.4), and by using the flat FRW metric (3.10), we find the usual continuity equation

$$\dot{\rho} + 3\left(\rho + p\right)H = 0. \tag{3.13}$$

Furthermore, substituting the expressions for energy density ρ and pressure density p from eqs. (3.5) and (3.6) we obtain the motion equation

$$\dot{B} = -2BH. \tag{3.14}$$

Clearly, the equations (3.11), (3.13) and (3.14) are not all independent equations. From equation (3.11) and (3.13) we also can obtain eq. (3.14). The source of the inflation is the strong magnetic fields in the early universe. By using the energy density ρ and pressure p in eq. (3.7) and eq. (3.8), and integrating the eq. (3.13), the evolution of the magnetic field B with respect to the scale factor a is obtained: $B(t) = B_0/a(t)^2$. It is noted that B_0 stands for $a(t) = 1$, where the inflation causes the increasing of the scale factor a, on the other hand the magnetic field decreases.

4   Perfect fluid description and dynamics of inflation

4.1   Slow-roll parameters

Following ref. [67], the field equations (3.11), (3.13) and (3.14) can be rewritten in the form of an effective perfect fluid as follows:

$$\frac{3}{\kappa^2}H(N)^2 = \rho_{\text{eff}}(N), \tag{4.1}$$

$$-\frac{2}{\kappa^2}H(N)\frac{dH(N)}{dN} = \rho_{\text{eff}}(N) + P_{\text{eff}}(N), \tag{4.2}$$

where the cosmic time derivatives have been expressed as derivatives with respect to the number of e-folds through $dN = Hdt$. In this way, we have that $N = \log(a(t)/a_i)$. This effective fluid description includes the energy densities and pressures coming from the non-liner magnetic monopole field and the non-minimal coupling term. Here the effective energy

density and the effective pressure densities take the following form:

$$\rho_{\text{eff}}(N) = \frac{X+1}{2\left[\beta(X+1)^2 + \kappa^2\lambda\left(3X-1\right)\right]}, \tag{4.3}$$

$$P_{\text{eff}}(N) = \left[\frac{1}{6\left(\beta(X+1)^2 + \kappa^2\lambda(3X-1)\right)^2}\right] \\ \times \left[\beta(X-3)(X+1)^2 + \kappa^2\lambda\left(X(10-9X)+3\right)\right], \tag{4.4}$$

where:

$$X(N) = \exp(4N)/(\beta B_0^2). \tag{4.5}$$

The equation of state (EoS) parameter of the total fluid is given by

$$P_{\text{eff}}(N) = -\rho_{\text{eff}}(N) + f(\rho_{\text{eff}}), \tag{4.6}$$

where the function $f(\rho_{\text{eff}})$ is given by:

$$f(\rho_{\text{eff}}) = -\frac{1}{12\kappa^2\lambda} - \frac{\rho_{\text{eff}}}{3}(16\beta\rho_{\text{eff}} - 3) \\ -3\kappa^2\lambda\rho_{\text{eff}}^2 \pm \left[\frac{1}{12\kappa^2\lambda} + \frac{5\rho_{\text{eff}}}{6}\right] \\ \times \left[4\kappa^2\lambda\rho_{\text{eff}}\left(\rho_{\text{eff}}\left(16\beta + 9\kappa^2\lambda\right) - 3\right) + 1\right]^{\frac{1}{2}} = \\ f(N) = \frac{2X\left[4\kappa^2\lambda + \beta(X+1)^2\right]}{3\left[\beta(X+1)^2 + \kappa^2\lambda(3X-1)\right]^2}. \tag{4.7}$$

Therefore, by using eqs. (4.6), (4.3), and (4.7), the effective EoS parameter reads as:

$$w_{\text{eff}}(N) \equiv \frac{P_{\text{eff}}(N)}{\rho_{\text{eff}}(N)} = -1 + \frac{f(\rho_{\text{eff}})}{\rho_{\text{eff}}(N)} = \\ = \frac{(X-3)(X+1)^2 + \kappa^2\alpha(X(10-9X)+3)}{3(X+1)^3 + 3\kappa^2\alpha(3X-1)(X+1)}, \tag{4.8}$$

where $\alpha \equiv \frac{\lambda}{\beta}$. During inflation $|f(\rho_{\text{eff}})/\rho_{\text{eff}}(N)| \ll 1$ and thus $w_{\text{eff}}(N) \approx -1$. In fact, from the above equation, we have that at early times, when $X \to 0$, the EoS parameter goes to -1, while for late-times, when $X \to \infty$, the EoS tends to $\frac{1}{3}$. Then, in the effective fluid description, the nonlinear magnetic monopole behaves as an effective cosmological constant at the very early universe, driving inflation and while for later times, it behaves as a radiation fluid. This novel feature implies that a separate reheating phase is avoided in comparison to scalar field inflation.

In order to study the dynamics of inflation, we introduce the slow-parameter $\varepsilon \equiv -\frac{1}{H(N)}\frac{dH(N)}{dN} = -\frac{3}{2}f(\rho_{\text{eff}})/\rho_{\text{eff}}$, which satisfies the condition $\varepsilon \ll 1$ during slow-roll inflation. By using eq. (4.2), ϵ has the following dependence in X:

$$\varepsilon = \frac{2X\left(X^2 + 2X + 4\alpha + 1\right)}{(X+1)\left(X^2 + (3\alpha+2)X - \alpha + 1\right)}. \tag{4.9}$$

It is interesting to mention that a very early times, when $X \to 0$, ε goes to zero, while in the limit $X \to \infty$, we have that ε becomes 2, which is the exact value for ε in a radiation-dominated universe, since the scale factor evolves as $a(t) \propto t^{1/2}$.

On the other hand, the end of inflation takes place when the condition $\varepsilon = 1$ is satisfied, such that, $X(t_{\text{end}}) = X_{\text{end}}$. Thus, from eq. (4.9) we obtain:

$$X_{\text{end}} = \kappa^2\alpha - \frac{1}{3} + \frac{4}{3U} - \frac{8\kappa^2\alpha}{U} + \frac{3\kappa^4\alpha^2}{U} + \frac{U}{3}, \tag{4.10}$$

where:

$$U = \left[8 + 36\kappa^2\alpha - 108\kappa^4\alpha^2 + 27\kappa^6\alpha^3 + 6\sqrt{3}S\right]^{1/3}, \tag{4.11}$$

$$S = \left[\kappa^2\alpha\left(16 - 72\kappa^2\alpha + 108\kappa^4\alpha^2 - 27\kappa^6\alpha^3\right)\right]^{1/2}. \tag{4.12}$$

For a sake of comparison, in the minimally coupled case, i.e, $\lambda = 0$ (or equivalently $\alpha = 0$) we find that $S = 0$, $U = 2/3$ and thus we have that $X_{\text{end}} = 1$.

4.2 Comparison with current observations

As a first approach, the perturbative analysis of our model can be also developed in the framework of an effective fluid description [67], where the scalar spectral index, the tensor-to-scalar ratio, and the running of the spectral index take the form:

$$n_s \simeq 1 - 6\frac{f(\rho_{\text{eff}})}{\rho_{\text{eff}}(N)}, \quad r \approx 24\frac{f(\rho_{\text{eff}})}{\rho_{\text{eff}}(N)}, \quad \alpha_s \approx -9\left(\frac{f(\rho_{\text{eff}})}{\rho_{\text{eff}}(N)}\right)^2, \tag{4.13}$$

where

$$\frac{f(\rho_{\text{eff}})}{\rho_{\text{eff}}(N)} = \frac{4X\left[(1+X)^2 + 4\kappa^2\alpha\right]}{(1+X)\left[(1+X)^2 + \kappa^2\alpha(3X-1)\right]}. \tag{4.14}$$

The form of eq. (4.13) is valid when the condition $|f(\rho_{\text{eff}})/\rho_{\text{eff}}(N)| \ll 1$ is satisfied during inflation. Then, r, n_s, and α_s for a given value of the X variable are certain functions of β and α. First, and having in mind that the radiation-dominated epoch after inflation takes place for very large values of X, we fix $X_* = 4 \times 10^2$, and consider the case $\beta = 1 \times 10^{-2}M_{pl}^{-4}$, being M_{pl} the reduced Planck mass. For this fixing of parameters we plot r versus n_s in the same plot with the allowed contour at the 68 and 95 % C.L. from the latest Planck data, as is shown in figure 1. The theoretical prediction lies inside the 95 % C.L. region when α takes values in the following range:

$$1.98 \times 10^5 M_{pl}^2 < \alpha < 7.74 \times 10^5 M_{pl}^2, \tag{4.15}$$

implying that the scalar spectral index lies inside the range:

$$0.968 < n_s < 0.972. \tag{4.16}$$

On the other hand, since $\alpha = \lambda/\beta$, then the constraint for the coupling λ becomes:

$$1.98 \times 10^3 M_{pl}^{-2} < \lambda < 7.74 \times 10^3 M_{pl}^{-2}. \tag{4.17}$$

In order to determine the prediction of this model regarding the running of the spectral index α_s, figure 2 shows the trajectories in the $n_s - dn_s/d\ln k$ plane for the allowed range for α already obtained, and we found that these values are close to -2×10^{-4}, being in agreement with current bound imposed by last data of Planck.

The smooth transition from inflation up to the radiation-dominated epoch at background level is shown in figure 3 and figure 4. In particular, figure 3 depicts the evolution
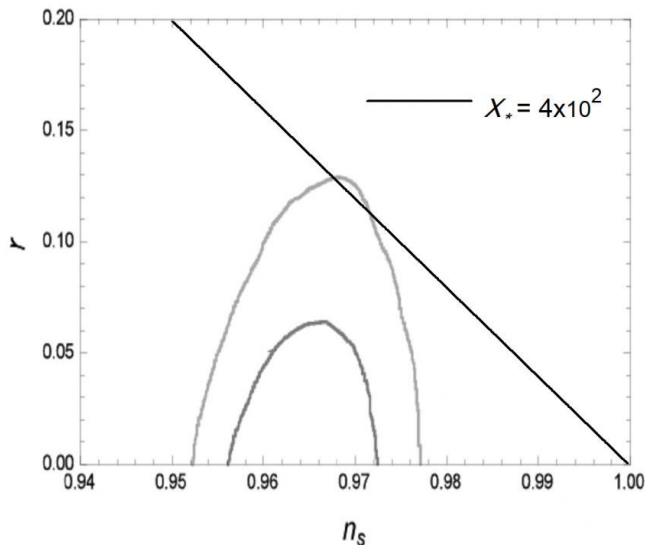
Figure 1. Allowed contours at the 68 and 95 % C.L., from the latest Planck data and theoretical predictions in the plane r versus n_s for the fluid description (4.13), by fixing $X_* = 4 \times 10^2$ and $\beta = 1 \times 10^{-2} M_{pl}^{-4}$.

of the effective EoS parameter w_{eff}, given by eq. (4.8), as function of X, for three different values of α within the allowed range found previously. In this way, it is demonstrated that the model interpolates between and effective cosmological constant driving inflation ($w_{\text{eff}} \simeq -1$) and a radiation fluid ($w_{\text{eff}} = 1/3$). On the other hand, figure 4 shows the evolution of the slow-roll parameter ε, given by eq. (4.9), which is also plotted against the X variable for three different values of α. We can see again that, at early times, an slow-roll inflationary epoch takes place ($\varepsilon \ll 1$) and, for late times, the evolution corresponds to a radiation-dominated universe ($\varepsilon = 2$).

As we have seen at perturbative level, the effective fluid description yields that our model becomes only marginally consistent with current observations. However, a more rigorous treatment of perturbations at linear model may be regarded as a further research.

In next subsection, we will use the current bound on the propagation speed of gravitational waves to put constraints on both parameters α and λ.

4.3 Complying with the gravitational waves constraints

By linearising the field equations around Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $h_{\mu\nu} \ll 1$, and for a perfect fluid matter source which has a Lagrangian density that satisfies $\mathcal{L}_m = -\rho \approx const$, one is led to the following wave equation for the tensor modes [35]

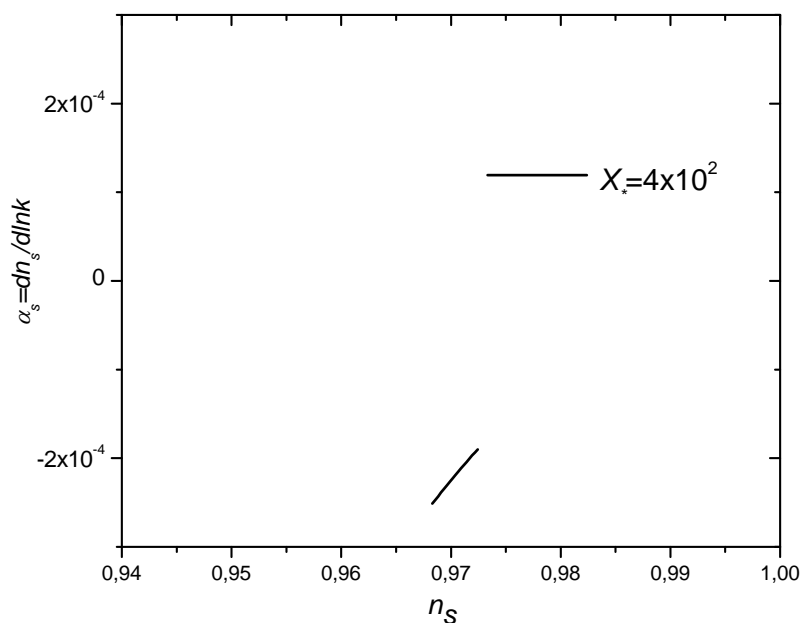$$\Box \left(h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h \right) = m_g^2 h_{\mu\nu}, \tag{4.18}$$

Figure 2. Plot of the running of the scalar spectral index $dn_s/d\ln k$ for the allowed range for α, by fixing $X_* = 4 \times 10^2$ and $\beta = 1 \times 10^{-2}M_{pl}^{-4}$.
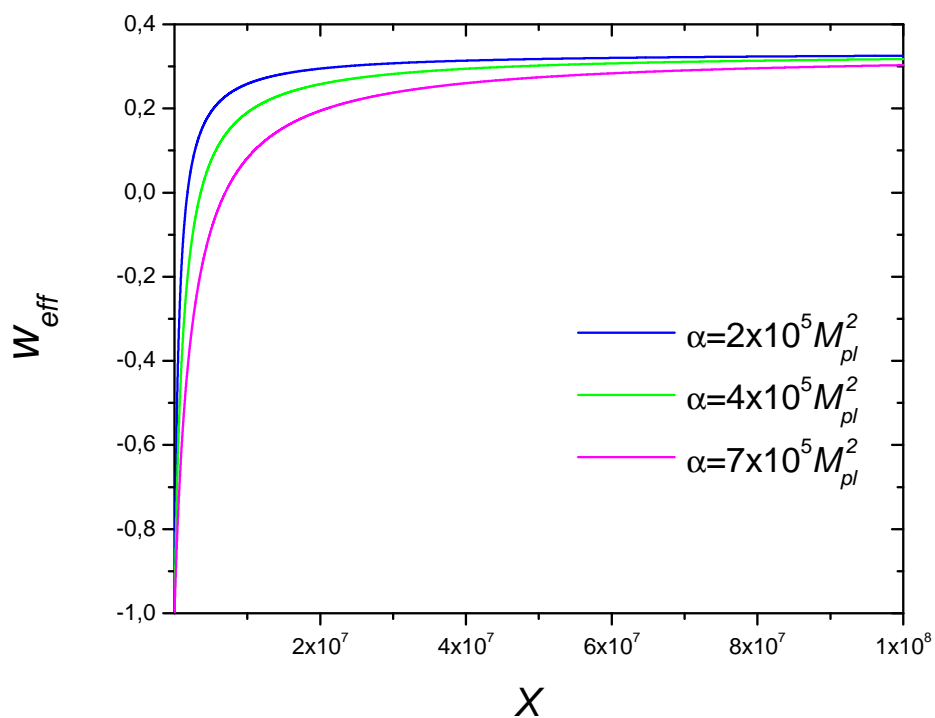

Figure 3. Plot of the effective EoS parameter w_{eff} against the X variable for three different values of α by fixing $\beta = 1 \times 10^{-2}M_{pl}^{-4}$.
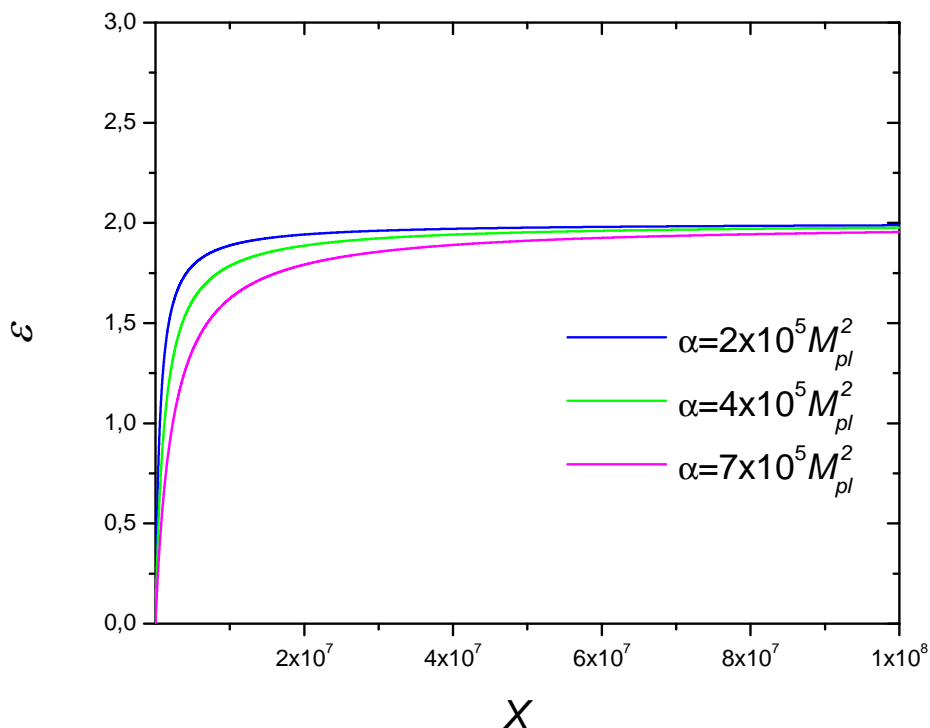
Figure 4. Plot of the slow-roll parameter ε against the X variable for three different values of α by fixing $\beta = 1 \times 10^{-2} M_{pl}^{-4}$.

where

$$m_g^2 = \frac{f_1(R_0) - 2f_2(R_0)\rho}{F_1(R_0) - 2F_2(R_0)\rho}, \tag{4.19}$$

is the squared mass of the graviton, $m_g < 7.7 \times 10^{-23} eV$ [68], and $R_0 = 0$ for the Minkowski background. The group velocity of the gravitational wave is calculated from the corresponding dispersion relation which gives $v_g^2 = 1 - m_g^2/2k^2$ [69]. Here, an extra longitudinal mode related to $\delta\rho$ is also present into the theory [35]. In the case of $f_1(R) = R$ and $f_2(R) = 1 + \lambda R$, one can see that

$$m_g^2 = -\frac{2\rho}{1 - 2\lambda\rho}. \tag{4.20}$$

We have focused on a nonlinear monopole magnetic field as matter source, whose Lagrangian density is given by (3.1). Since the solution found for the magnetic field scales as $B(t) = B_0/a(t)^2$, and $\mathcal{F} = B(t)^2/2 = B_0^2/2a(t)^4$, thus, for small values of the scale factor, $a(t) \to 0$, one obtains $\mathcal{L}_{NMM} = -\rho \approx -1/(2\beta)$ being that β is a constant. Therefore, by substituting this expression for energy density ρ into eq. (4.20), we find $m_g^2 = -\beta^{-1}/(1 - \alpha)$, and hence

$$\alpha > 2.84 \times 10^{-1} M_{pl}^2, \tag{4.21}$$

for $\beta = 1 \times 10^{-2} M_{pl}^{-4}$.

If we compare eq. (4.21) with the constraint in eq. (4.15), obtained from constraints on the inflationary observables, one can see that the large values required for the parameter α and λ guarantee a very small mass of the graviton in agreement with observations [68], and

therefore, also a propagation speed of gravitational waves very close to the speed of light with a high precision [27, 30].

In the opposed limit, when the scale factor tends to larger values, $a(t) \to \infty$, the effective coupling term between curvature and matter behaves as a^{-6}, and hence, the matter sector, which comes to behave as radiation, decouples from curvature, and trivially satisfying all the gravitational waves constraints of GR.

5   Concluding remarks

It has been studied the inflationary dynamics of a nonlinear monopole magnetic field which is nonminimally coupled to curvature. In the context of nonminimally coupled $f(R)$ gravity theories, the gravitational sector is taken to have the Einstein-Hilbert form, whereas that the matter Lagrangian density, that is, the nonlinear monopole magnetic field, is linearly coupled to curvature. We have used an effective fluid description in order to calculate the effective EOS and the slow-roll parameters. In this framework, the nonlinear magnetic monopole field behaves as an effective cosmological constant at the very early Universe, driving inflation and while for later times, it behaves as a radiation fluid. This novel feature implies that a separate reheating phase is avoided in comparison to scalar field inflation.

After, by using the most recent observational data from Planck collaboration, we set the limits on the parameter of the nonminimal coupling, and the quotient of the nonminimal coupling and the nonlinear monopole magnetic scales. First, we have used the theoretical prediction that we have found coherent with the 95 % C.L. region as shown in figure 1 for the value of the α:

$$1.98 \times 10^5 M_{pl}^2 < \alpha < 7.74 \times 10^5 M_{pl}^2, \tag{5.1}$$

and then the constraint for the coupling λ becomes:

$$1.98 \times 10^3 M_{pl}^{-2} < \lambda < 7.74 \times 10^3 M_{pl}^{-2}. \tag{5.2}$$

Then we have checked our model by running the spectral index α_s, in figure 2 to show the trajectories in the $n_s - dn_s/d\ln k$ plane for the allowed range for α, hence these values are close to -2×10^{-4}, being in agreement with current bound imposed by last data of Planck.

We have seen that at perturbative level, the effective fluid description yields that our model becomes only marginally consistent with current observations. However, a more rigorous treatment of perturbations at linear model can be regarded as a further research.

Second, we have obtained the constraints for the nonminimal coupling parameter α using the current bound on the propagation speed of gravitational waves to as follows:

$$\alpha > 2.84 \times 10^{-1} M_{pl}^2, \tag{5.3}$$

for $\beta = 1 \times 10^{-2} M_{pl}^{-4}$. By comparing eqs. (4.15) and (4.21), we have concluded that the large values required for the parameter α, and λ, obtained from constraints on inflationary observables, are consistent with a very small mass of the graviton in agreement with observations [68], and therefore, also verifying a propagation speed of gravitational waves very close to the speed of light with a high precision in agreement with the recent gravitational wave data GW170817 of LIGO and Virgo [27, 30].

Linde [70] and Vilenkin [71] discussed the inflation could occur in the cores of topological defects, where the scalar field is forced to stay near the maximum of its potential. However in our paper, we have used nonlinear magnetic monopole field nonminimally coupled to

curvature as source of cosmic inflation. From conceptual point of view, the class of models treated in our work, has the advantage that it involves only electromagnetic fields, and it does not presuppose the existence of exotic entities such as scalar fields or another speculative modified matter models. It is worth noting that, the only fundamental scalar field which has been observed in nature is the Higgs field, but we do not have certainty if it can be identified as the inflaton once that it alone cannot generate inflation, being required a nonminimal coupling to gravity and a Higgs mass m_H greater than 230 Gev which is outside the widely accepted upper bound $m_H \sim 180$ Gev [72, 73].

Acknowledgments

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