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A PARTICULAR THIN-SHELL WORMHOLE

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Using a black hole with scalar hair, we construct a scalar thin-shell wormhole (TSW) in 2+1 dimensions by applying the Visser cut and paste technique. The surface stress, which is concentrated at the wormhole throat, is determined using the Darmois–Israel formalism. Using various gas models, we analyze the stability of the TSW. The stability region is changed by tuning the parameters l and u. We note that the obtained TSW originating from a black hole with scalar hair could be more stable with a particular value of the parameter l, but it still requires exotic matter.

Keywords: thin-shell wormhole, stability, Darmois–Israel formalism, scalar hair, black hole

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1. Introduction

In 1988, Morris and Thorne [1] devised traversable wormholes, which are solutions of Einstein's gravitation equations. They are cosmic shortcuts that connect two points of the Universe by a throatlike geometry. But they violate one or more of the so-called energy conditions: the weak energy condition (WEC), null energy condition (NEC), and strong energy condition (SEC) (see, e.g., [2]–[4]). Most physicists therefore agree that wormholes require exotic matter—a kind of antigravity—to keep their throat (the narrowest point) open [5]. In contrast, some physicists studying this subject claim that wormholes, such as the thin-shell wormhole (TSW), can be supported by normal matter [6], [7].

At first, Visser [8] proposed a method for constructing TSWs by applying the Israel junction conditions [9]. It was shown that the amount of exotic matter [10] around the throat can be minimized with a suitable choice of the wormhole geometry. In the literature, there are now many studies following Visser's prescription focused on constructing TSWs described in arbitrary (lower or higher) dimensions (see, e.g., [11]–[33]). Here, we consider the scalar-hair black hole (SHBH) in 2+1 dimensions that is a solution of the Einstein–Maxwell theory with a self-interacting scalar field described by the Liouville potential $V(\phi)$ [34]. Using the standard cut-and-paste technique, we then construct a TSW and test its stability with different physical gas states.

Our main motivation in constructing a TWH is to minimize the exotic matter, which is in general the main source for supporting the throat. Here, we focus on the stability of the SHBH space–time in 2+1 dimensions because this black hole depends on two variables and we can obtain stable solutions by choosing them.

This paper is organized as follows. In Sec. 2, we briefly describe the SHBH in (2+1)-dimensional

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geometry. In Sec. 3, we first introduce the basic concepts related to the TSW and then in Sec. 4 apply various gas models to the equation of state (EoS) to study its stability. We present conclusions in Sec. 5.

2. The SHBH space–time

In this section, we briefly review the SHBH [34]. The Einstein–Maxwell gravity minimally coupled to a scalar field ϕ is described by the action

$$S = \int \sqrt{-g}\big(R - 2\,\partial_\mu\phi\,\partial^\mu\phi - F^2 - V(\phi)\big)d^3r, \tag{1}$$

where R denotes the Ricci scalar, $F = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant, and $V(\phi)$ denotes the scalar ϕ potential. The SHBH is a solution of action (1) found in [34] as

$$ds^2 = -f(r)\,dt^2 + \frac{4r^2\,dr^2}{f(r)} + r^2\,d\theta^2, \tag{2}$$

where the metric function is

$$f(r) = \frac{r^2}{l^2} - ur. \tag{3}$$

Here, u and l are constants, and the event horizon of BH (2) is located at $r_{\rm h} = u\ell^2$. It is clear that this BH does not have a asymptotically flat geometry. Metric (2) can alternatively be written in the form

$$ds^2 = -\frac{r}{\ell^2}(r - r_{\rm h})\,dt^2 + \frac{4r\ell^2\,dr^2}{r - r_{\rm h}} + r^2\,d\theta^2. \tag{4}$$

The singularity located at $r = 0$ can be seen best by checking the Ricci and Kretschmann scalars:

$$R = -\frac{2r + r_{\rm h}}{4r^3\ell^2}, \tag{5}$$

$$K = \frac{4r^2 - 4r_{\rm h}r + 3r_{\rm h}^2}{16r^6\ell^4}. \tag{6}$$

The scalar field and potential are respectively given by [34]

$$\phi = \frac{\log r}{\sqrt{2}}, \tag{7}$$

$$V(\phi) = \frac{\lambda_1 + \lambda_2}{r^2}, \tag{8}$$

where $\lambda_{1,2}$ are constants. The corresponding Hawking temperature of the BH (see, e.g., [35]) is

$$T_{\rm H} = \frac{1}{4\pi}\frac{\partial f}{\partial r}\bigg|_{r=r_{\rm h}} = \frac{1}{8\pi\ell^2}, \tag{9}$$

which is constant. Radiation with a constant temperature is a well-known isothermal process. We note that Hawking radiation of the linear dilaton black holes exhibits a similar isothermal behavior [35]–[42].

3. Stability of a TSW

In this section, we consider two identical copies of the SHBHs with

$$M^{\pm} = (x|r \geq 0), \quad r \geq a,$$

and the manifolds bounded by hypersurfaces M^{+} and M^{-}. To obtain a single manifold $M = M^{+} + M^{-}$, we match them at the surface of the junction

$$\Sigma^{\pm} = (x|r = a),$$

where the boundaries Σ are given. On shell, we can choose

$$ds^2 = -d\tau^2 + a^2(\tau)\, d\theta^2 \tag{10}$$

as the space–time, where τ represents the proper time [20]. Setting the coordinates $\xi^i = (\tau, \theta)$, we write the extrinsic curvature formula connecting the two sides of the shell in the simple form [26]

$$K_{ij}^{\pm} = -n_{\gamma}^{\pm}\left(\frac{\partial^2 x^{\gamma}}{\partial \xi^i\, \partial \xi^j} + \Gamma^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial \xi^i}\frac{\partial x^{\beta}}{\partial \xi^j}\right), \tag{11}$$

where the unit normals $((n^{\gamma} n_{\gamma} = 1)$ are

$$n_{\gamma}^{\pm} = \pm\left|g^{\alpha\beta}\frac{\partial H}{\partial x^{\alpha}}\frac{\partial H}{\partial x^{\beta}}\right|^{-1/2}\frac{\partial H}{\partial x^{\gamma}}, \tag{12}$$

where $H(r) = r - a(\tau)$. With the metric functions, the nonzero components of $n_{\gamma}^{\pm}$ become

$$n_t = \mp 2a\dot{a}, \tag{13}$$

$$n_r = \pm 2\sqrt{\frac{al^2(4\dot{a}^2 l^2 a - l^2 u + a)}{(l^2 u - a)}}, \tag{14}$$

where the dot denotes the derivative with respect to τ. The nonzero components of extrinsic curvature (11) are then written as

$$K_{\tau\tau}^{\pm} = \mp\frac{\sqrt{-al^2}(8\dot{a}^2 l^2 a + 8\ddot{a} l^2 a^2 - l^2 u + 2a)}{4a^2 l^2\sqrt{-4\dot{a}^2 l^2 a - l^2 u + a}}, \tag{15}$$

$$K_{\theta\theta}^{\pm} = \pm\frac{1}{2a^{3/2} l}\sqrt{4\dot{a}^2 l^2 a - l^2 u + a}. \tag{16}$$

Because K_{ij} is not continuous around the shell H [26], we use the Lanczos equation [43]–[45]

$$S_{ij} = -\frac{1}{8\pi}([K_{ij}] - [K] g_{ij}), \tag{17}$$

where K is the trace of K_{ij}, $[K_{ij}] = K_{ij}^{+} - K_{ij}^{-}$, and S_{ij} is the energy–momentum tensor at the junction, which in the general case has the form [11], [26]

$$S^i_j = \mathrm{diag}(\sigma - p), \tag{18}$$

where p is the surface pressure and σ is the surface energy density. Because of the circular symmetry, we have

$$K^i_j = \begin{pmatrix} K^\tau_\tau & 0 \\ 0 & K^\theta_\theta \end{pmatrix}. \tag{19}$$

From Eqs. (17) and (18), we therefore obtain the surface pressure and the surface energy density [26].

Using the cut-and-paste technique, we can excise the interior regions $r < a$ of geometry (10) and link its exterior parts. But there exists a bounce (deduced from the extrinsic curvature components at the surface $r = a$) that is related to the energy density and pressure:

$$\sigma = -\frac{1}{8\pi a^{3/2} l}\sqrt{4\dot{a}^2 l^2 a - l^2 u + a}, \tag{20}$$

$$p = \frac{1}{16\pi a^{3/2} l}\frac{8\dot{a}^2 l^2 a + 8\ddot{a} l^2 a^2 - l^2 u + 2a}{\sqrt{4\dot{a}^2 l^2 a - l^2 u + a}}. \tag{21}$$

Consequently, the energy and pressure quantities in the static case ($a = a_0$) become

$$\sigma_0 = -\frac{1}{8\pi a_0^{3/2} l}\sqrt{-l^2 u + a_0}, \tag{22}$$

$$p_0 = \frac{1}{16\pi a_0^{3/2} l}\frac{-l^2 u + 2a_0}{\sqrt{-l^2 u + a_0}}. \tag{23}$$

If $\sigma \geq 0$ and $\sigma + p \geq 0$, then the WEC is satisfied. In addition, the condition $\sigma + p \geq 0$ is the NEC. Moreover, the SEC depends on satisfying $\sigma + p \geq 0$ and $\sigma + 2p \geq 0$. It is obvious from Eq. (24) that negative energy density violates the WEC, and we consequently need exotic matter for constructing the TSW. We note that the total matter supporting the wormhole is given by [46]

$$\Omega_\sigma = \int_0^{2\pi} (\rho\sqrt{-g})\big|_{r=a_0}\, d\phi = 2\pi a_0 \sigma(a_0) = -\frac{1}{4a_0^{1/2}|l|}\sqrt{-l^2 u + a_0}. \tag{24}$$

We investigate the stability of such a wormhole using a linear perturbation in which the EoS is written as

$$p = \psi(\sigma), \tag{25}$$

where $\psi(\sigma)$ is an arbitrary function of σ. We introduce the energy conservation equation as [26]

$$S^i_{j;i} = -T_{\alpha\beta}\,\frac{\partial x^\alpha}{\partial \xi^j}\, n^\beta, \tag{26}$$

where $T_{\alpha\beta}$ is the bulk energy–momentum tensor. We can hence rewrite Eq. (26) in terms of the pressure and energy density:

$$\frac{d}{d\tau}(\sigma a) + \psi\,\frac{da}{d\tau} = -\dot{a}\sigma. \tag{27}$$

From this equation, we obtain

$$\sigma' = -\frac{1}{a}(2\sigma + \psi), \tag{28}$$

and its second derivative yields

$$\sigma'' = \frac{2}{a^2}(\tilde{\psi} + 3)\left(\sigma + \frac{\psi}{2}\right), \tag{29}$$

where prime and tilde denote the respective derivatives with respect to a and σ. The equation of motion for the shell in the general case is

$$\dot{a}^2 + V = 0, \tag{30}$$

where the effective potential V is found from Eq. (22) as

$$V = \frac{1}{4l^2} - \frac{u}{4a} - 16a^2\sigma^2\pi^2. \tag{31}$$

In fact, Eq. (30) is just the equation of the oscillatory motion in which the stability around the equilibrium point $a = a_0$ depends on satisfying the condition $V''(a_0) \geq 0$. Using Eqs. (30) and (31), we finally obtain

$$V'' = -\frac{1}{2a^3}\left[64\pi^2 a^5(\sigma\sigma')' + 4\sigma'\frac{\sigma}{a} + \frac{\sigma^2}{a^2}) + u\right]\Big|_{a=a_0} \tag{32}$$

or, equivalently,

$$V'' = \frac{1}{2a^3}\left\{-64\pi^2 a^3[(2\psi' + 3)\sigma^2 + \psi(\psi' + 3)\sigma + \psi^2] - u\right\}\Big|_{a=a_0}. \tag{33}$$

The equation of motion of the throat for a small perturbation becomes [47]–[49]

$$\dot{a}^2 + \frac{V''(a_0)}{2}(a - a_0)^2 = 0.$$

We note that under the condition $V''(a_0) \geq 0$, the TSW is stable, and the motion of the throat is oscillatory with the angular frequency $\omega = \sqrt{V''(a_0)/2}$.

4. Some models of the EoS supporting the TSW

In this section, we use particular gas models—linear barotropic gas (LBG) [47], [50], Chaplygin gas (CG) [51], [52], generalized Chaplygin gas (GCG) [53], and logarithmic gas (LogG [20]—to investigate the stability of the TSW.

4.1. Stability analysis of the TSW in the LBG model. The EoS of the LBG [47], [50] is given by

$$\psi = \varepsilon_0\sigma, \tag{34}$$

and hence

$$\psi'(\sigma_0) = \varepsilon_0, \tag{35}$$

where ε_0 is a constant parameter. Changing the values of l and u in Eq. (35), we illustrate the stability regions for the TSW in terms of ε_0 and a_0, as shown in Fig. 1.

4.2. Stability analysis of the TSW in the CG model. The EoS of the CG that we consider is given by [51]

$$\psi = \varepsilon_0\left(\frac{1}{\sigma} - \frac{1}{\sigma_0}\right) + p_0, \tag{36}$$

and we naturally obtain

$$\psi'(\sigma_0) = -\frac{\varepsilon_0}{\sigma_0^2}. \tag{37}$$

Substituting Eq. (37) in Eq. (35), we plot the stability regions for the TSW supported by the CG in Fig. 2.

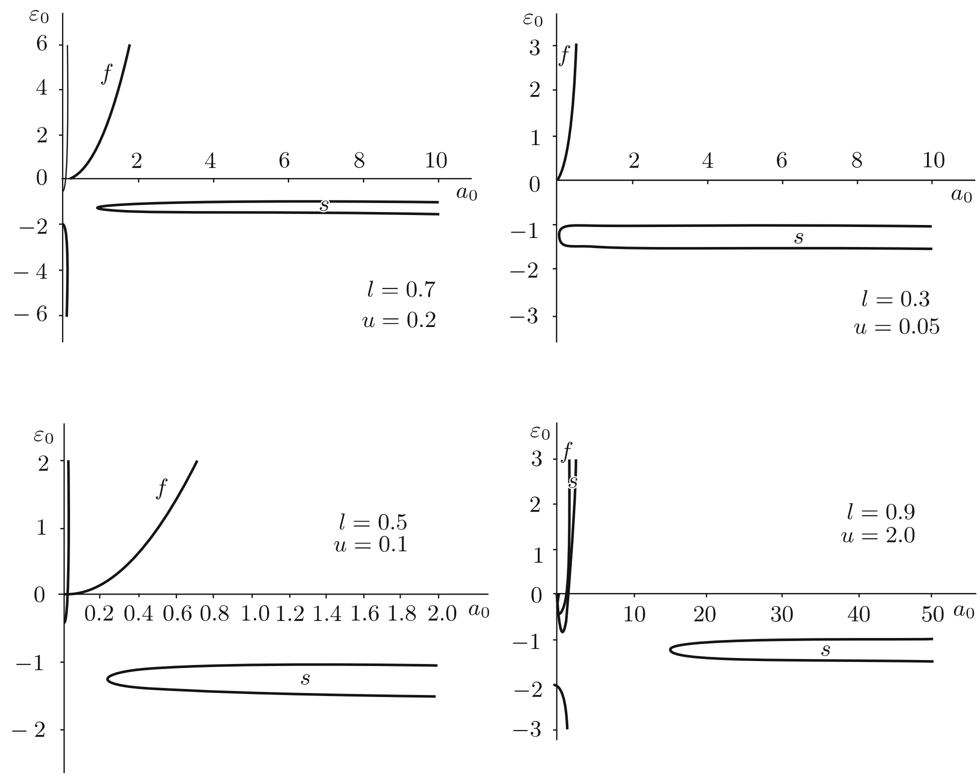

Fig. 1. Stability regions of the TSW in the LBG model for different values of l and u.

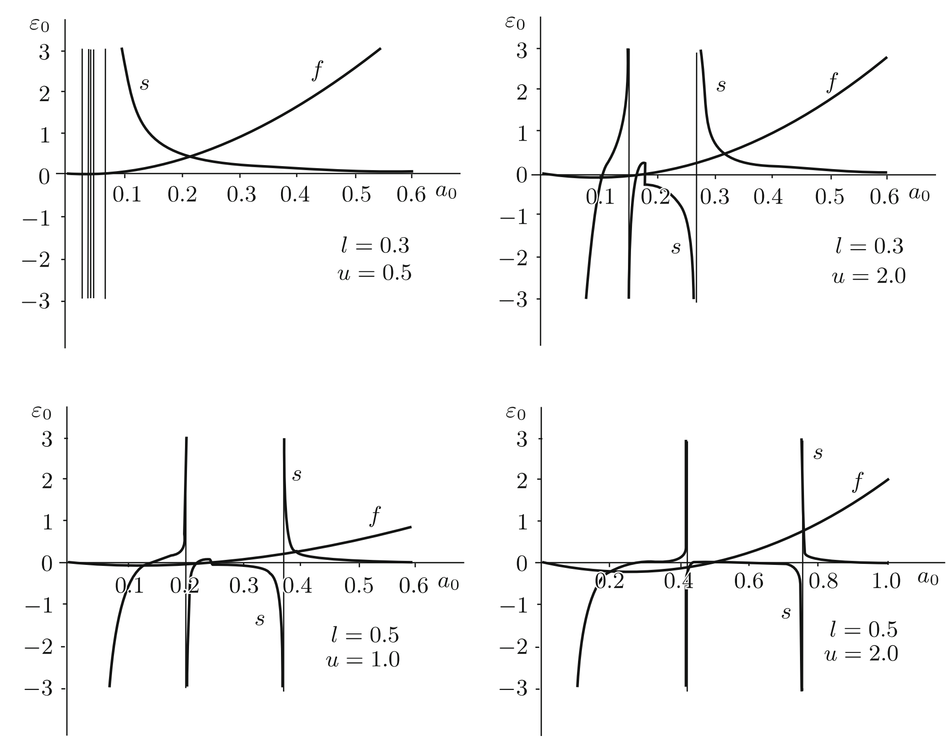

Fig. 2. Stability regions of the TSW in the CG model for different values of l and u.

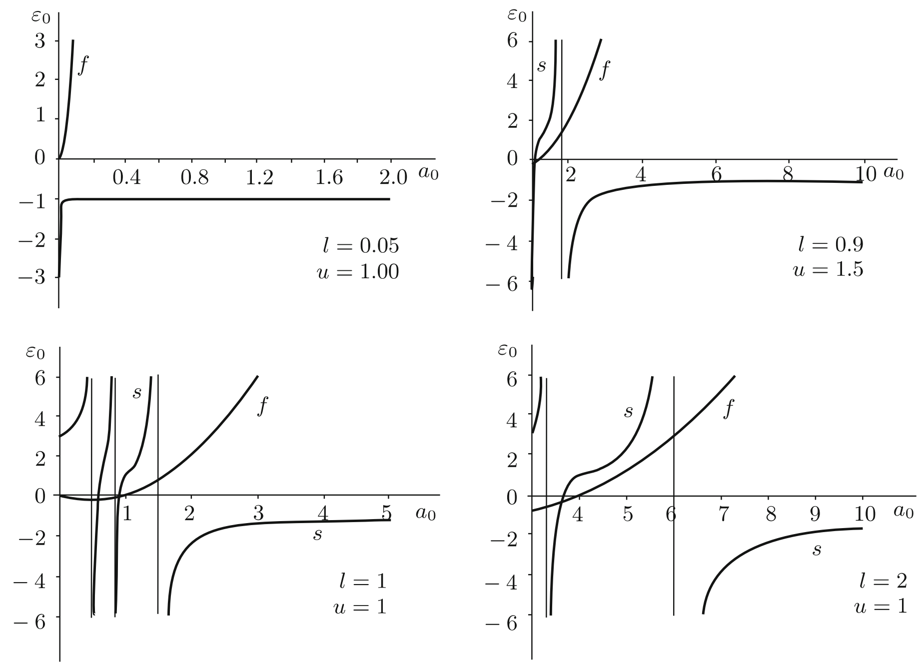

Fig. 3. Stability regions of the TSW in the GCG model for different values of l and u.

4.3. Stability analysis of the TSW in the GCG model. We use the EoS of the GCG [53]

$$\psi = p_0\left(\frac{\sigma_0}{\sigma}\right)^{\varepsilon_0}, \tag{38}$$

whence we obtain

$$\psi'(\sigma_0) = -\varepsilon_0\frac{p_0}{\sigma_0}. \tag{39}$$

In Fig. 3, we show the stability regions of a TSW supported by a GCG that are obtained by substituting Eq. (39) in Eq. (35).

4.4. Stability analysis of the TSW in the LogG model. In our final example, we choose the EoS for the LogG in the form (see [20])

$$\psi = \varepsilon_0 \log\frac{\sigma}{\sigma_0} + p_0, \tag{40}$$

which leads to

$$\psi'(\sigma_0) = \frac{\varepsilon_0}{\sigma_0}. \tag{41}$$

In Fig. 4, we show the stability regions of a TSW supported by a LogG that are obtained by substituting the expression presented above in Eq. (35).

5. Conclusion

We have constructed a TSW by gluing two copies of a SHBH by the cut-and-paste procedure. For this, we used the fact that the throat radius must be greater than the event horizon of the given metric: $a_0 > r_{\mathrm{h}}$. We applied the EoSs for the LBG, CG, GCG, and LogG representing the exotic matter located at the throat. The stability analysis then reduces to checking that the second derivative of the effective

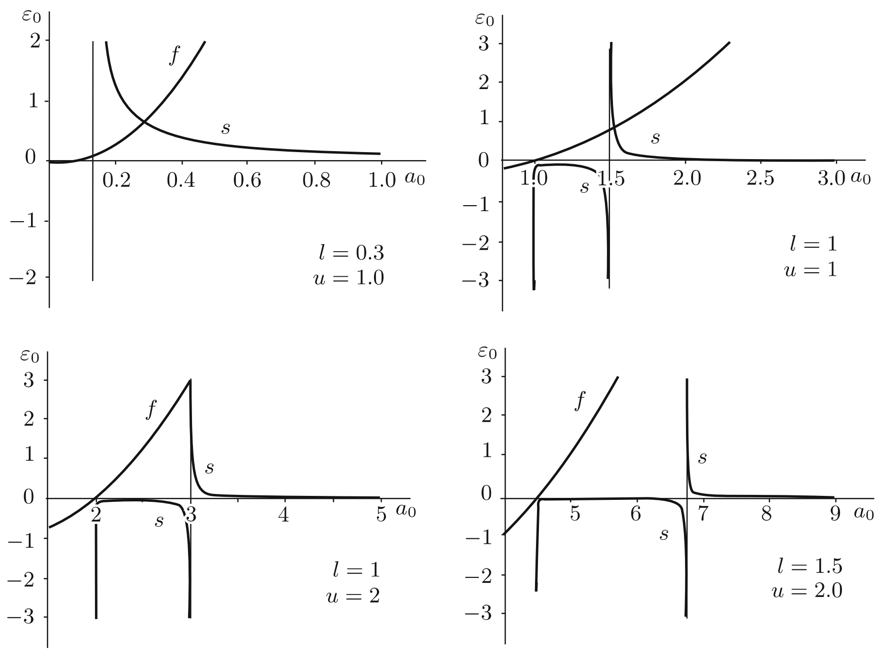

Fig. 4. Stability regions of the TSW in the LogG model for different values of l and u.

potential is positive at the throat radius a_0: $V''(a_0) \geq 0$. In all cases, we managed to find the stability regions in terms of the throat radius a_0 and the constant parameter ε_0 connected with the considered EoS. The problem of the angular perturbation is outside the scope of this paper. We therefore only considered a linear perturbation, but we plan to study the angular perturbation in our future continuation of this study. This will be done in the near future.

One of the most relevant topics in theoretical physics is the relation between an Einstein–Rosen (ER) bridge (wormhole) [54] and an Einstein–Podolsky–Rosen (EPR) bridge [55] (a synonym for entanglement [56], [57]). In our opinion, another open problem here is nature of the relation between the TSW and the EPR wormhole. Is it possible to solve the exotic matter problem of the TSW using the EPR wormhole, or vice versa? Are there any exotic forces between pairs of EPR wormholes? All these question remain open and await their solutions, and they should be adequately studied. Our next project is to add one more piece to this big puzzle.

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