Effect of nonlinear electrodynamics on the weak field deflection angle by a black hole

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In this work we investigate the weak deflection angle of light from an exact black hole within nonlinear electrodynamics. First, we calculate the Gaussian optical curvature using the optical spacetime geometry. With the help of modern geometrical methods popularized by Gibbons and Werner, we examine the deflection angle of light from an exact black hole. To do this, we determine the optical Gaussian curvature and apply the Gauss-Bonnet theorem to the optical metric and calculate the leading terms of the deflection angle in the weak-limit approximation. Furthermore, we also study the plasma medium's effect on weak gravitational lensing by an exact black hole. Hence, we determine the effect of nonlinear electrodynamics on the deflection angle in a weak gravitational field.

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I. INTRODUCTION

In 1783, accepting the corpuscular theory of light suggested by Newton which hypothesized that light is comprised of small discrete particles, John Michell proposed the presence of dark stars. Michell posted a letter to the *Philosophical affair of the Royal Society of London* [1] wherein he logically said that these small, discrete particles of light emitted by a star are slowed by the star's gravitational field, and he believed that it may be within reach to determine a star's mass be measuring its brightness. However, the gravitational pull of a star may be so strong that even light could not escape from it; this type of star was called a dark or invisible star. Michell evaluated that this might be the situation for a star with a mass 500 times greater than that of the Sun. Michell likewise declared that we may identify dark stars by studying binary star systems, yet here only one star would need to be observed. Michell's thought was ignored for over 100 years, since it was accepted that gravity could not be associated with light.

Moreover, in 1915, Einstein's theory of general relativity (GR) predicted that a gravitational lens [a concentration of matter (for example, a group of galaxies) located between the light source and the observer] could deflect the light from the source as the light travelled to the viewer. This effect is called gravitational lensing, a theory later proven by experiment in 1919 [2,3]. The topic of gravitational lensing received considerable theoretical attention and a number of observational phenomena related with the deflection of light rays by gravity were identified. The theory of gravitational lensing mostly involves geometrical optics in vacuum and uses the idea of the deflection angle. The essential assumption is the approximation of a weak photon deflection angle. General relativity explained that a light beam moving close to a circular body of mass M with a huge impact parameter b is deflected by a small angle,

$$\Theta = \frac{2R_S}{b} = \frac{4M}{b}, \qquad G = c = 1. \tag{1}$$

This interpretation is solid if $b \succeq R_S$, where $R_S = 2M$ is the Schwarzschild radius of the gravitating body. The deflection angle (1) is typically called the "Einstein angle." In most astrophysical circumstances associated with gravitational lensing, the weak-deflection approximation is valid. Just like the deflection angles, the directions of photons in vacuum do not rely on the light's frequency, and thus gravitational lensing in vacuum is neutral.

It was John Wheeler who coined the term black hole (BH) and introduced the concept of wormholes [4]. Since the experimental investigation of the deviation of light in 1919, various observations of the gravitational lensing have been made for BHs as well as other astrophysical objects [5–10].

Gravitational lensing is a helpful instrument of astrophysics [11] and astronomy. In gravitational lensing light beams from distant stars and galaxies are deflected by an object, such as a planet, BH, or dark matter [12,13]. The discovery of dark matter filaments [14] with the help of

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weak deflection is an extremely relevant topic since it is very helpful in studying the structure of the Universe [15]. From a hypothetical viewpoint, new techniques have been proposed to compute deflection angles. In 2008, Gibbons and Werner (GW) devised another plan to calculate the deflection angle of photons [16]. Gibbons and Werner imagined that both the light source and the observer lie in an asymptotically Minkowski region. Then, they applied the Gauss-Bonnet theorem (GBT) to a optical space, which is characterized by the optical metric [16]. In the GBT, we can utilize a space D_R bounded by the light ray and a circular boundary curve C_R that is situated at a focal point where the photon beam meets the light source and the observer. It is expected that both the light source and observer are at a coordinate length R from the focal point. In the weak-field approximation the GBT is given in terms of the optical metric as [16]

$$\int\int_{D_R} \mathcal{K} dS + \oint_{\partial D_R} \kappa dt + \Sigma_i \theta_i = 2\pi \mathcal{X}(D_R),$$

where $\mathcal{K}$ denotes the optical Gaussian curvature and dS denotes an areal component. Subsequently, by utilizing the Euler characteristic $\mathcal{X}(D_R) = 1$ and the jump angles $\Sigma_i \theta_i = \pi$, the deflection angle is calculated by assuming that the system obeys the straight-line approximation,

$$\alpha = -\int_0^\pi \int_{\frac{b}{r \sin\phi}}^\infty \mathcal{K} dS,$$

where the deflection angle is denoted by Θ. A short time later, Werner expanded this strategy for stationary BHs [17]. Next, Ishihara *et al.* [18] demonstrated that it is possible to calculate the deflection angle for finite distances (large impact parameter) as GW just calculated the deflection angle of the BH's spacetime for an observer in an asymptotically flat region in the weak-field limit using the optical geometry. Recently, Crisnejo and Gallo examined the deflection of light within a plasma medium [19]. Abdujabbarov *et al.* studied the gravitational lensing of BHs in the presence of plasma [20] and also observed the effect of plasma on the shadow of wormholes and BHs [21–24]. Turimov *et al.* [25] also checked the behavior of gravitational lensing in the presence of plasma. Moreover, Chakrabarty *et al.* [26] and Atamurotov *et al.* [27] also studied a plasma medium's effect on gravitational lensing and the shadow of a black hole. Moreover, Hensh *et al.* [28] calculated the gravitational lensing of Kehagias-Sfetsos compact objects in the presence of a plasma medium.

The GBT was later used to study weak gravitational lensing by BHs, cosmic strings, and wormholes [29–62].

The primary point of this paper is to explore the impact of the NLE on the deflection angle of an exact BH and utilize the GBT, wherein the deviation of light becomes a global effect. Since we just center the nonsingular field outer of a light beams, we mostly examine the gravitational singularities within general relativity. Here, the density clearly becomes infinite at the origin of the BH, which mimics the conditions during the big bang. In the theory of GR, spacetime singularities give rise to various issues, both scientific and physical [63,64]. By utilizing the NLE, it is possible to resolve these singularities by calculating a regular BH solution [65–70]. Recently, Kruglov suggested another model of NLE with two parameters β and γ, where the particular scope of magnetic field, the unitary conditions, and causality are fulfilled [71]. Furthermore, Aliev *et al.* demonstrated the impact of a magnetic field on a BH spacetime [72,73].

The rest of this paper is organized as follows. In Sec. II, we briefly describe an exact BH and compute its optical metric and the Gaussian optical curvature. In Sec. III, the deflection angle of light utilizing the GBT is computed for an exact BH. In Sec. IV, we graph the behavior of the deflection angle in nonplasma medium. In Sec. V, we examine the effect of a plasma medium on gravitational lensing. In Sec. VI, we graph the behavior of the deflection angle in the presence of a plasma medium. Finally, in Sec. VII we discuss our conclusions.

II. EXACT OPTICAL METRIC WITH NONLINEAR ELECTRODYNAMICS

The action that describes nonlinear electrodynamics (NLE) minimally coupled to gravity is characterized as follows [74]:

$$S = \frac{1}{16\pi} \int \sqrt{-g}(R + K(\psi)) d^4x, \tag{2}$$

where

$$\psi = F_{\mu\nu} F^{\mu\nu}, \qquad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu.$$

Here, R is the Ricci scalar, A_μ is the Maxwell field, g is the determinant of the metric, and $K(\psi)$ is defined as a function of ψ. The field equations are calculated as follows:

$$G_{\mu\nu} = -2K_{,\psi} F_{\mu\lambda} f_\nu^\lambda + \frac{1}{2} g_{\mu\nu} K, \qquad K_{,\psi} \equiv \frac{dK}{d\psi}, \tag{3}$$

and

$$\nabla_\mu (K_{,\psi} F^{\mu\nu}) = 0. \tag{4}$$

We work in the framework of a static and spherically symmetric spacetime, which can generally be written as

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega_2^2,$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$. As the spacetime is static and spherically symmetric, the nonvanishing A_μ is written as

$$A_0 = \phi(r),$$

and ψ is

$$\psi = -2\dot{\phi}^2$$

by using the approximation $A_\mu \to A_\mu + \nabla_\mu \mathcal{X}$. So, we get the Einstein equations and the derived Maxwell equation for the spherically symmetric spacetime,

$$-\frac{\dot{U}\dot{f}}{f} - \frac{2U\ddot{f}}{f} + \frac{1}{f^2} - \frac{U\dot{f}^2}{f^2} = 2K_{,\psi}\dot{\phi}^2 + \frac{1}{2}K, \qquad (5)$$

$$-\frac{\dot{U}\dot{f}}{f} + \frac{1}{f^2} - \frac{U\dot{f}^2}{f^2} = 2K_{,\psi}\dot{\phi}^2 + \frac{1}{2}K, \qquad (6)$$

$$\frac{\dot{U}\dot{f}}{f} + \frac{U\ddot{f}}{f} + \frac{1}{2}\ddot{U} = -\frac{1}{2}K, \qquad (7)$$

$$(f^2 K_{,\psi}\dot{\phi})^{\cdot} = 0. \qquad (8)$$

A dot denotes a derivative with respect to r. From the above equations, we derive the values $G_0^0 = \rho$, $G_1^1 = p_r$, and $G_2^2 = p_\theta$. Here, Eq. (8) is the equation of motion for the Maxwell field.

Presently, the measurement of the static and spherically symmetric spacetime with the nonlinear electrodynamics field is defined as [74]

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega_2^2, \qquad (9)$$

where

$$U(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha,$$
$$d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2,$$

where M is the black hole mass, Q is the charge, and α is the coupling constant. Now, we insert the value of U into Eq. (9) to obtain

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha\right)dt^2 \\ &+ \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha\right)^{-1} dr^2 \\ &+ r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \end{aligned} \qquad (10)$$

By accepting that the light source and observer lie in the in equatorial plane similarly direction of the null photon is in a similar plane having $(\theta = \frac{\pi}{2})$. Now, for null geodesics we put $ds^2 = 0$ and we get the following optical metric:

$$\begin{aligned} dt^2 = &\frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha\right)^2} \\ &+ \frac{r^2 d\phi^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha}. \end{aligned} \qquad (11)$$

Now, the optical metric is written in terms of the new coordinates $r^\star$ as

$$dt^2 = \bar{g}_{ab} dx^a dx^b = dr^{\star 2} + f^2(r^\star) d\phi^2, \qquad (12)$$

where

$$r^\star = \frac{r}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha},$$
$$f(r^\star) = \frac{r}{\sqrt{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2\alpha^2}{3} + 2Q\alpha\right)}}. \qquad (13)$$

Here we see that (a, b) is converted into (r, ϕ) and its determinant is $\det \bar{g}_{ab} = \frac{1}{f(r^\star)^2}$. Now, by using Eq. (12) the nonzero Christoffel symbols are defined as

$$\Gamma^{r^\star}_{\phi\phi} = -f(r^\star)f'(r^\star) \quad \text{and} \quad \Gamma^{\phi}_{r^\star\phi} = \frac{f'(r^\star)}{f(r^\star)},$$

and the only nonvanishing Riemann tensor for the optical curvature is given as $R_{r^\star\phi r^\star\phi} = -kf^2(r^\star)$, where $R_{r^\star\phi r^\star\phi} = g_{r^\star r^\star} R^{r^\star}_{\phi r^\star \phi}$. Now, the Gaussian optical curvature is written as

$$\mathcal{K} = \frac{R_{r^\star\phi r^\star\phi}}{g_{r^\star\phi}} = -\frac{f''(r^\star)}{f(r^\star)} = \frac{-1}{f(r^\star)} \frac{d^2 f(r^\star)}{dr^{\star 2}}. \qquad (14)$$

With the help of the previous equation, the intrinsic Gaussian optical curvature denoted by $\mathcal{K}$ can be written in terms of r as

$$\mathcal{K} = \frac{-1}{f(r^\star)}\left[\frac{dr}{dr^\star}\frac{d}{dr}\left(\frac{dr}{dr^\star}\right)\frac{df}{dr} + \frac{d^2 f}{dr^2}\left(\frac{dr}{dr^\star}\right)^2\right]. \qquad (15)$$

Finally, we calculate the relevant Gaussian optical curvature for an exact BH by putting Eq. (13) into Eq. (15),

$$\begin{aligned} \mathcal{K} = &\frac{-2M}{r^3}\left(1 - \frac{3M}{2r}\right) + \frac{3Q^2}{r^4}\left(1 + \frac{2Q^2}{3r^2}\right) - \frac{4MQ\alpha}{r^3} - \frac{6MQ^2}{r^5} \\ &- \frac{2Q^2\alpha}{r^2}\left(\alpha - \frac{3Q}{r^2}\right) + \alpha^2\left(\frac{2M}{r} - \frac{1}{3} - \frac{2Q\alpha}{3}\right), \end{aligned} \qquad (16)$$

which can be written as

$$\mathcal{K} = \frac{-2M}{r^3} + \frac{3Q^2}{r^4} - \frac{4MQ\alpha}{r^3} + \mathcal{O}(M^{-2}). \qquad (17)$$

III. DEFLECTION ANGLE OF AN EXACT BLACK HOLE WITHIN NONLINEAR ELECTRODYNAMICS

Now, with the help of Gauss-Bonnet theorem we derive the deflection angle of an exact black hole in the presence of nonlinear electrodynamics. We apply the Gauss-Bonnet theorem to the region $\mathcal{D}_R$, stated as [16]

$$\iint_{\mathcal{D}_R} \mathcal{K} dS + \oint_{\partial \mathcal{D}_R} k dt + \sum_i \epsilon_i = 2\pi \mathcal{X}(\mathcal{D}_R), \quad (18)$$

where the Gaussian curvature is denoted by $\mathcal{K}$ and the geodesic curvature is denoted by k, where $k = \bar{g}(\nabla_{\dot{\gamma}}\dot{\gamma}, \ddot{\gamma})$ in such a way that $\bar{g}(\dot{\gamma}, \dot{\gamma}) = 1$, where $\ddot{\gamma}$ is the representation for the unit acceleration vector and ϵ_i is the corresponding exterior angle at the ith vertex. As $R \to \infty$, both jump angles become $\pi/2$ and we obtain $\theta_O + \theta_S \to \pi$. The Euler characteristic is $\mathcal{X}(\mathcal{D}_R) = 1$, as $\mathcal{D}_R$ is nonsingular. Therefore, we get

$$\iint_{\mathcal{D}_R} \mathcal{K} dS + \oint_{\partial \mathcal{D}_R} k dt + \epsilon_i = 2\pi \mathcal{X}(\mathcal{D}_R), \quad (19)$$

where $\epsilon_i = \pi$ proves that $\gamma_{\bar{g}}$ and the total jump angle is a geodesic, since the Euler characteristic number denoted by $\mathcal{X}$ is 1. As $R \to \infty$, the only interesting part to be calculated is $k(C_R) = |\nabla_{\dot{C}_R} \dot{C}_R|$. Since the geodesic curvature's radial component is given by [16]

$$(\nabla_{\dot{C}_R} \dot{C}_R)^r = \dot{C}_R^\phi \partial_\phi \dot{C}_R^r + \Gamma^{r^\star}_{\phi\phi} (\dot{C}_R^\phi)^2, \quad (20)$$

for large R, $C_R := r(\phi) = R = \text{const}$. Hence, Eq. (20) becomes $(\dot{C}_R^\phi)^2 = \frac{1}{f^2(r^\star)}$. Remembering that $\Gamma^{r^\star}_{\phi\phi} = -f(r^\star) f'(r^\star)$, this becomes

$$(\nabla_{\dot{C}_R^r} \dot{C}_R^r)^r \to \frac{+1}{R}. \quad (21)$$

Hence, we see that the topological defect is not involved in the geodesic curvature. So, $k(C_R) \to R^{-1}$, but with the help of the optical metric (12) we can write it as $dt = R d\phi$. Hence, we obtain

$$k(C_R) dt = \frac{1}{R} R d\phi. \quad (22)$$

Combining all of the above results, we have

$$\iint_{\mathcal{D}_R} \mathcal{K} ds + \oint_{\partial \mathcal{D}_R} k dt \overset{R\to\infty}{=} \iint_{S_\infty} \mathcal{K} dS + \int_0^{\pi+\Theta} d\phi. \quad (23)$$

A light ray in the weak-deflection limit at zeroth order is defined as $r(t) = b/\sin\phi$. So, with the help of Eqs. (17) and (24), the deflection angle is defined as [16]

$$\Theta = -\int_0^\pi \int_{b/\sin\phi}^\infty \mathcal{K} \sqrt{\det \bar{g}} dr^\star d\phi, \quad (24)$$

where

$$\sqrt{\det \bar{g}} = r\left(1 - \frac{3M}{r} + \frac{3Q^2}{2r^2} + 3Q\alpha\right) dr. \quad (25)$$

After putting the leading-order terms of the Gaussian curvature (17) into Eq. (24), the deflection angle is defined as

$$\Theta \approx \frac{4M}{b} - \frac{3\pi Q^2}{4b^2} + \frac{20MQ\alpha}{b}. \quad (26)$$

IV. GRAPHICAL ANALYSIS FOR A NONPLASMA MEDIUM

In this section we perform a graphical analysis of the deflection angle. We also talk about the physical importance of the above-mentioned plots and observe the effect of the coupling constant α, impact parameter b, and BH charge Q on the deflection angle.

A. Deflection angle Θ with respect to coupling constant α

Figures 1(a) and 1(b) show the behavior of Θ with respect to α by varying Q with $b = 5M$ and varying b with $Q = 5M$, respectively.

(1) In Fig. 1(a), we observe that Θ gradually decreases for large values of Q.

(2) In Fig. 1(b), we observe that Θ gradually increases for large values of b.

B. Deflection angle Θ with respect to impact parameter b

Figures 2(a) and 2(b) show the behavior of Θ with respect to b for fixed α and varying Q.

(1) In Figs. 2(a) and 2(b), we observe that Θ gradually decreases for both small and large values of Q.

Figures 2(c) and 2(d) show the behavior of Θ with respect to b for fixed BH charge and varying coupling constant.

(1) In Figs. 2(c) and 2(d), we observe that Θ also gradually decreases for both large and small values of the coupling constant.

C. Deflection angle Θ with respect to BH charge Q

Figures 3(a) and 3(b) show the behavior of Θ with respect to Q by varying b with $\alpha = 5/M$ and varying α with $b = 5M$, respectively.

(1) In Fig. 3(a), we observe that Θ increases exponentially for large values of the impact parameter, and the deflection angle rapidly increases for $1M < b < 5M$.

(2) In Fig. 3(b), we observe that Θ gradually decreases for large values of the coupling constant.

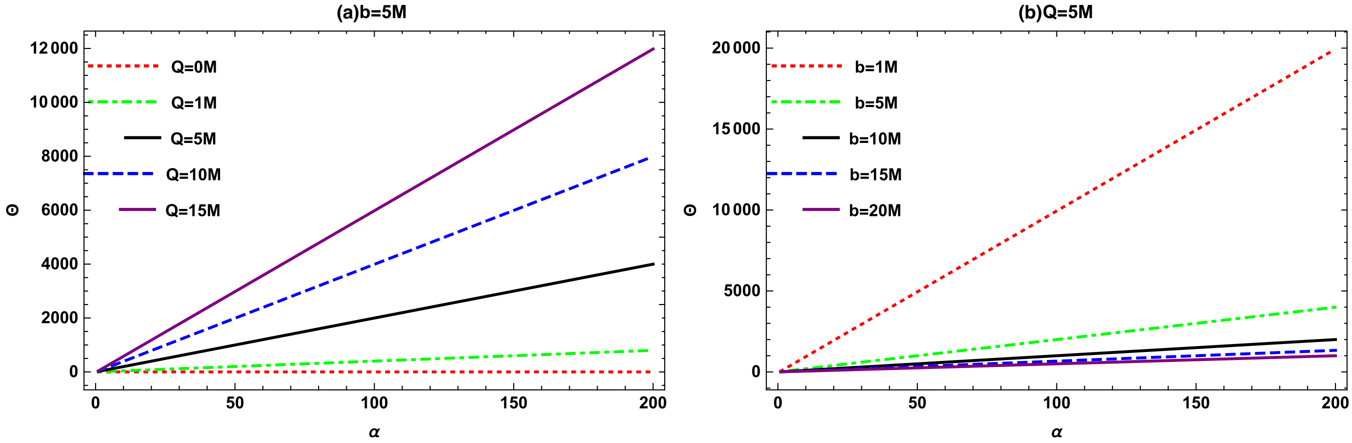

FIG. 1. Relation between Θ and α.

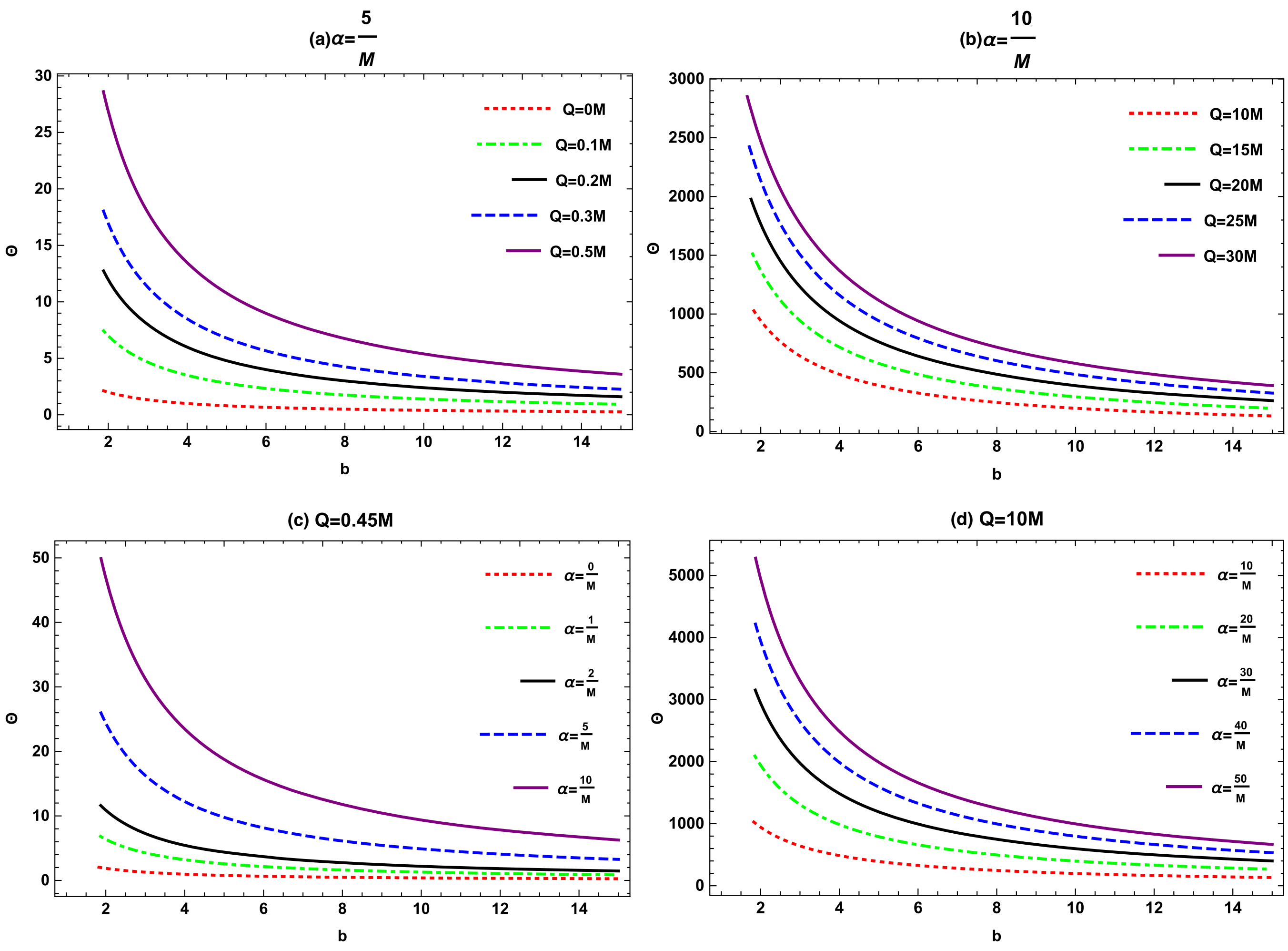

FIG. 2. Relation between Θ and b.

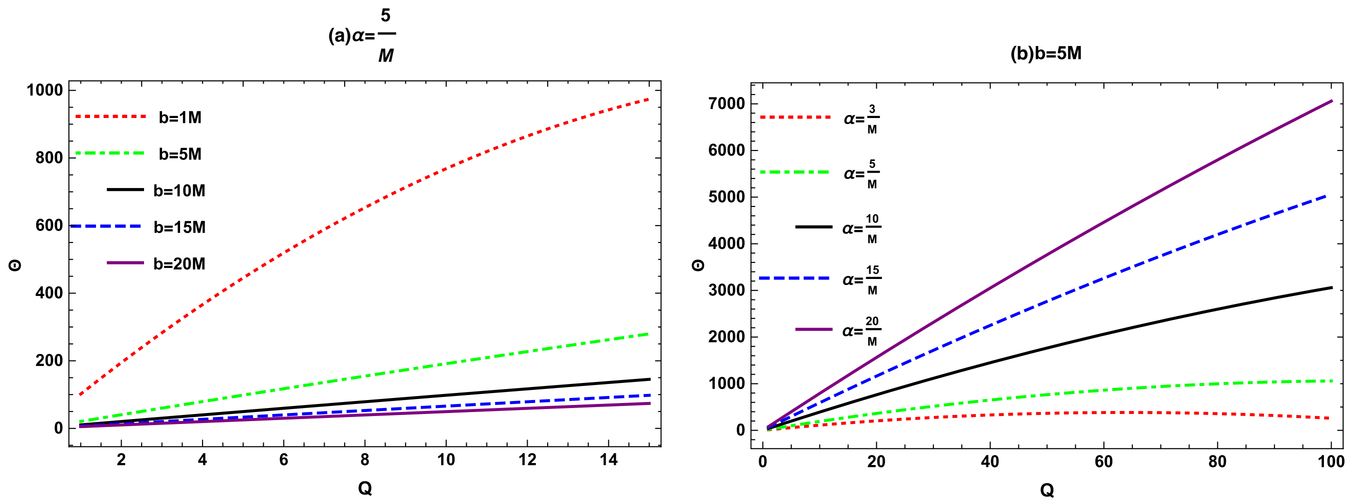

FIG. 3. Relation between Θ and Q.

V. EFFECT OF PLASMA ON GRAVITATIONAL LENSING

Here we examine the impact of a plasma medium on the gravitational lensing of an exact BH. Let us consider an exact BH imbued with a plasma described by the refractive index n [19]

$$n^2(r,\omega(r)) = 1-\frac{\omega_e^2(r)}{\omega_\infty^2(r)}. \tag{27}$$

The refractive index for this case reads

$$n(r) = \sqrt{1-\frac{\omega_e^2}{\omega_\infty^2}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{r^2\alpha^2}{3}+2Q\alpha\right)}, \tag{28}$$

where the metric function is defined by

$$ds^2 = -U(r)dt^2+\frac{1}{U(r)}dr^2+r^2d\Omega_2^2 \tag{29}$$

and

$$U(r) = 1-\frac{2M}{r}+\frac{Q^2}{r^2}-\frac{r^2\alpha^2}{3}+2Q\alpha.$$

By accepting that both the light source and observer lies in the tropical plane similarly direction of the null photon is in a similar plane having ($\theta=\frac{\pi}{2}$). Now, for null geodesics we put $ds^2=0$ and we get the following optical metric [19]:

$$dt^2 = g_{lm}^{\rm opt}dx^l dx^m = n^2\left[\frac{dr^2}{U^2(r)}+\frac{r^2d\phi^2}{U(r)}\right], \tag{30}$$

with determinant $g_{lm}^{\rm opt}$,

$$\sqrt{g^{\rm opt}} = r\left(1-\frac{\omega_e^2}{\omega_\infty^2}\right)+M\left(3-\frac{\omega_e^2}{\omega_\infty^2}\right)-\frac{Q^2}{2r}\left(3-\frac{\omega_e^2}{\omega_\infty^2}\right) - Q\alpha r\left(3-\frac{\omega_e^2}{\omega_\infty^2}\right). \tag{31}$$

With the help of Eq. (30), we can define the nonzero Christoffel symbols as

$$\Gamma^0_{00} = \left(1+\frac{\omega_e^2A}{\omega_\infty^2}\right)\left[-A'A^{-1}\left(1-\frac{\omega_e^2A}{\omega_\infty^2}\right)-\frac{A'\omega_e^2}{2\omega_\infty^2}\right],$$

$$\Gamma^1_{10} = \left(1+\frac{\omega_e^2A}{\omega_\infty^2}\right)\left[r^{-1}(1-\frac{\omega_e^2A}{\omega_\infty^2}-\frac{A'A^{-1}}{2}\left(1-\frac{\omega_e^2A}{\omega_\infty^2}\right)-\frac{A'\omega_e^2}{2\omega_\infty^2}\right],$$

and

$$\Gamma^0_{11} = \left(1+\frac{A\omega_e^2}{\omega_\infty^2}\right)\left[-rA\left(1-\frac{A\omega_e^2}{\omega_\infty^2}\right) + \frac{r^2A'}{2}\left(1-\frac{A\omega_e^2}{\omega_\infty^2}\right)+\frac{r^2A}{2}\frac{A'\omega_e^2}{\omega_\infty^2}\right].$$

The Gaussian curvature in terms of the curvature tensor can be determined as

$$\mathcal{K} = \frac{R_{r\phi r\phi}(g^{\rm opt})}{\det(g^{\rm opt})}, \tag{32}$$

and with the help of Eq. (32) the Gaussian curvature is written as

$$\mathcal{K} = \frac{M}{r^3}\left(-2-\frac{\omega_e^2}{\omega_\infty^2}+\frac{2\omega_e^4}{\omega_\infty^4}\right)+\frac{2MQ^2}{r^5}\left(1-\frac{17\omega_e^2}{\omega_\infty^2}+\frac{5\omega_e^4}{\omega_\infty^4}\right) - \frac{4MQ\alpha}{r^3}\left(1+\frac{\omega_e^2}{\omega_\infty^2}-3\frac{\omega_e^4}{\omega_\infty^4}\right)+\mathcal{O}(M^{-2}). \tag{33}$$

With the help of the Gauss-Bonnet theorem we calculate the deflection angle in order to relate it with a nonplasma medium. To calculate the angle in the weak-field region, we use the straight-line approximation and that $r = \frac{b}{\sin\phi}$ at zeroth order,

$$\Theta = -\lim_{R\to 0}\int_0^{\pi}\int_{\frac{b}{\sin\phi}}^{R}\mathcal{K}dS. \tag{34}$$

With the help of Eq. (23), the deflection angle of light in a plasma medium is defined as

$$\Theta = \frac{4M}{b} - \frac{2M\omega_e^2}{b\omega_\infty^2} - \frac{6M\omega_e^4}{b\omega_\infty^4} - \frac{3Q^2\pi}{4b^2} + \frac{3Q^2\pi\omega_e^4}{4b^2\omega_\infty^4} + \frac{4MQ\alpha}{b} + \frac{2MQ\alpha\omega_e^2}{b\omega_\infty^2}. \tag{35}$$

VI. GRAPHICAL ANALYSIS FOR A PLASMA MEDIUM

In this section we perform a graphical analysis of the deflection angle in the presence of a plasma medium. Here, we take $M = 1$, $\frac{\omega_e}{\omega_\infty} = 10^{-1}$, and vary the impact parameter, coupling constant, and BH charge to obtain these graphs.

A. Deflection angle with respect to coupling constant

Figures 4(a) and 4(b) show the behavior of Θ with respect to α by varying Q with $b = 5M$ and varying b with $Q = 5M$, respectively.

(1) In Fig. 4(a), we observe that Θ gradually decreases for small values of the BH charge Q and graph exhibits a positive slope.

(2) In Fig. 4(b), we observe that Θ exponentially increases for large values of b.

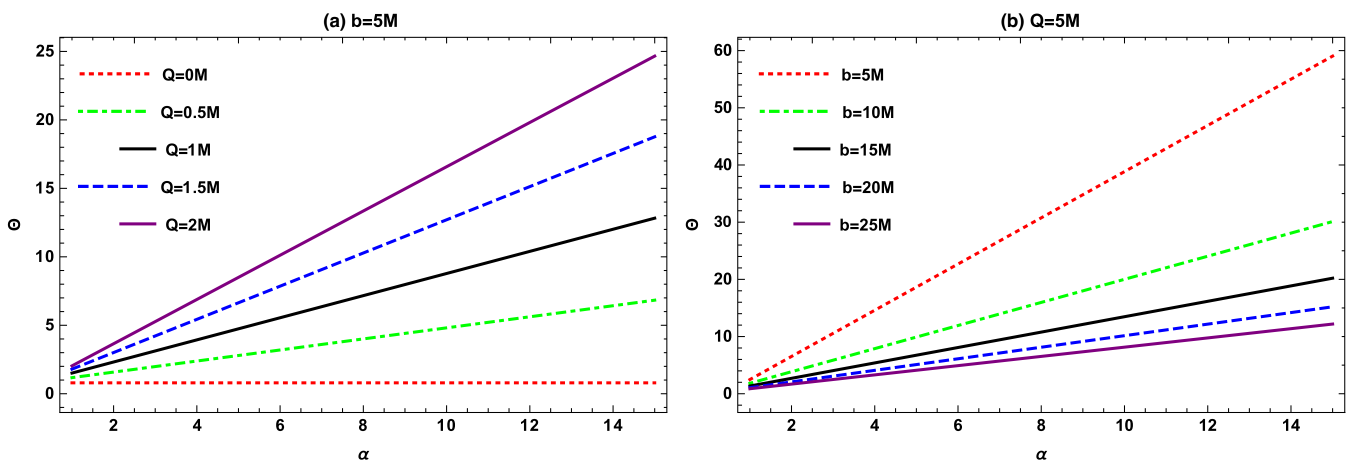

FIG. 4. Relation between Θ and α.

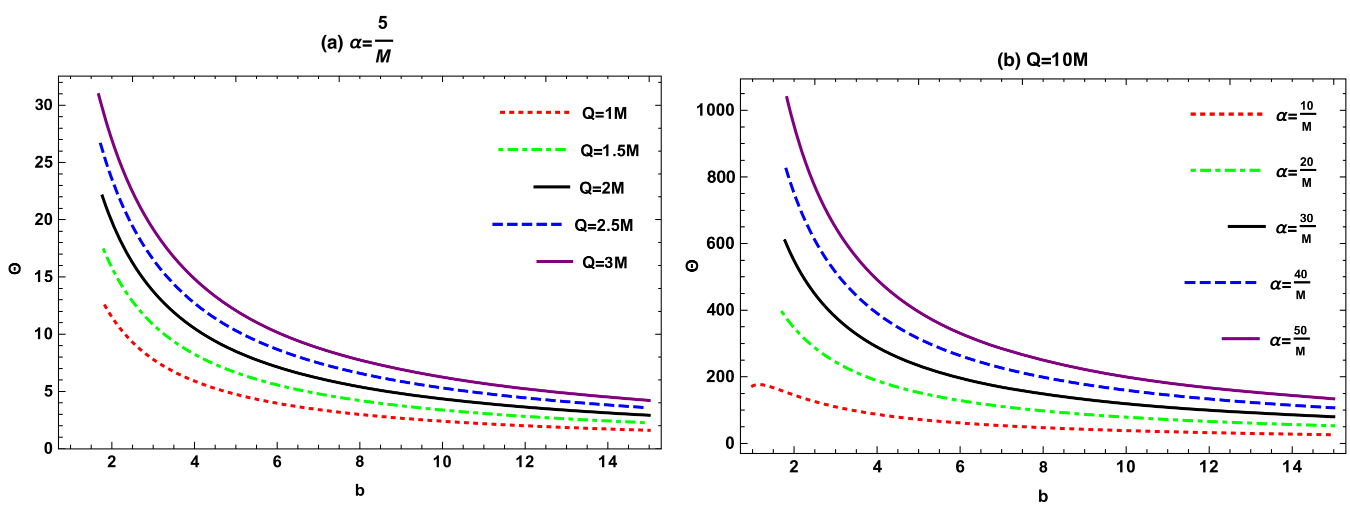

FIG. 5. Relation between Θ and b.

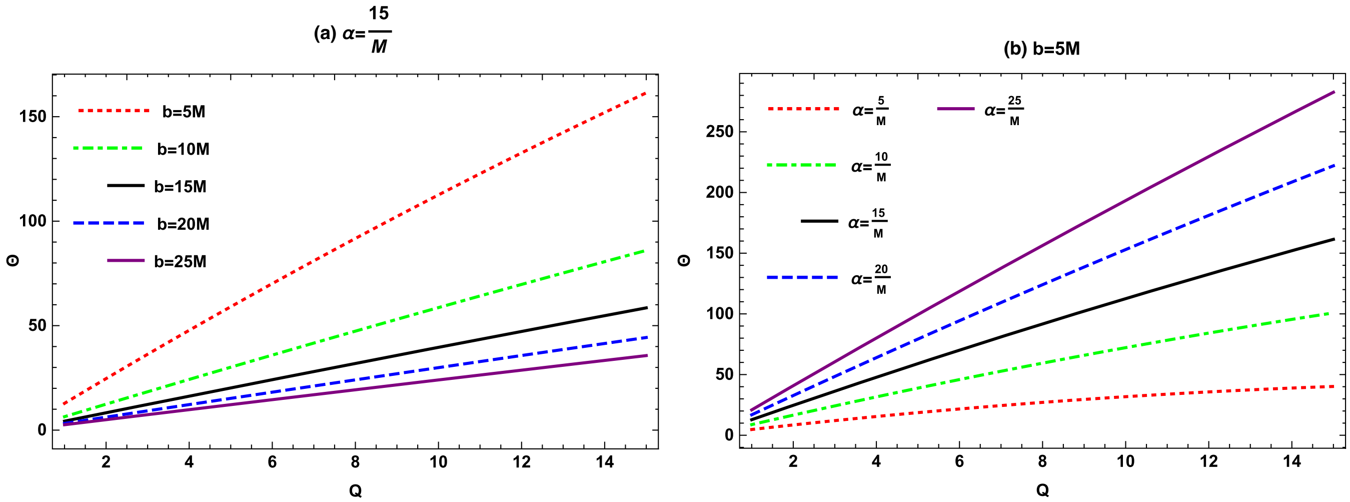

FIG. 6. Relation between Θ and Q.

B. Deflection angle Θ with respect to impact parameter b

Figures 5(a) and 5(b) show the behavior of Θ with respect to b by varying Q with $\alpha = 5/M$ and varying α with $Q = 10M$, respectively.

(1) In Fig. 5(a), we observe that Θ gradually decreases for small values of Q and then goes to positive infinity.

(2) In Fig. 5(b), we observe that Θ gradually decreases for large values of α and then goes to positive infinity.

C. Deflection angle with respect to charge Q

Figures 6(a) and 6(b) show the behavior of the deflection angle with respect to the BH charge for a fixed coupling constant and varying impact parameter and a fixed impact parameter and varying coupling constant, respectively.

(1) In Fig. 6(a), we observe that Θ gradually increases for large values of the impact parameter and the graph has a positive slope.

(2) In Fig. 6(b), we observe that Θ gradually increases for large values of the coupling constant and the graph has a positive slope.

VII. SUMMARY

In this work we have calculated the deflection angle for an exact BH in the framework of NLE. To do this, we used the Gauss-Bonnet theorem and determined the deflection angle for an exact BH with NLE. We utilized the GBT and found the deflection angle of photons by integrating over a domain outside the impact parameter, that represent that gravitational lensing is a global impact and is a useful asset to analyze most of the singularities of BH. In this calculation, we obtained the deflection angle of light by an exact BH in the weak-field limit by utilizing the GBT. Hence, the deflection angle (26) is expressed as

$$\Theta \approx \frac{4M}{b} - \frac{3\pi Q^2}{4b^2} + \frac{20MQ\alpha}{b} + \mathcal{O}(M^{-2}).$$

By setting $Q = 0$ in the above equation, our proposed deflection angle reduces to the Schwarzschild deflection angle up to first order. We also plotted the behavior of the deflection angle for an exact BH in the background of NLE. Furthermore, we computed the deflection angle of photons by an exact BH with NLE in a plasma medium. The deflection angle of photons in the presence of a plasma medium is defined as

$$\Theta = \frac{4M}{b} - \frac{2M\omega_e^2}{b\omega_\infty^2} - \frac{6M\omega_e^4}{b\omega_\infty^4} - \frac{3Q^2\pi}{4b^2} + \frac{3Q^2\pi\omega_e^4}{4b^2\omega_\infty^4} + \frac{4MQ\alpha}{b} + \frac{2MQ\alpha\omega_e^2}{b\omega_\infty^2}.$$

By neglecting the plasma impact ($\frac{\omega_e}{\omega_\infty} \to 0$), Eq. (35) reduces to Eq. (26).

We observed the behavior of the deflection angle with respect to the impact parameter b, coupling constant α, and BH charge Q. The results of our deflection angle analysis can be summarized as follows.

Deflection angle with respect to impact parameter:

(1) We observed that the deflection angle gradually decreases for large values of Q.

(2) We also observed that the deflection angle gradually decreases for large values of α, which shows the stability of our proposed deflection angle.

Deflection angle with respect to coupling constant:

(1) We only observed stable behavior of the deflection angle by an exact BH for $0M < Q \leq 2M$.

(2) The obtained deflection angle increases for increasing impact parameter, which indicates stable behavior.

Deflection angle with respect to BH charge:

(1) We observed that the deflection angle exponentially increases for large values of the impact parameter.

(2) We also observed that there is a direct relation between the deflection angle and the coupling constant.

To close, we observed that in the presence of a plasma medium the deflection angle of a BH decreases as compared to that for a BH in vacuum. Compared with Refs. [20–28], we also confirmed the result that the deflection angle decreases more in a plasma medium compared to vacuum cases. The authors of Ref. [28] showed that the radius of the shadow of a black hole increases when the plasma parameter increases; thus, in the future we will study the effect of a plasma medium on the shadow of a nonelectrodynamic black hole.

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