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Effect of the GUP on the Hawking radiation of black hole in 2 + 1 dimensions with quintessence and charged BTZ-like magnetic black hole

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In this paper, we investigate the Hawking radiation process by using the quantum tunneling phenomenon of massive spin-1 (W-bosons) and spin-0 particles by the black hole in $2+1$ dimensions surrounded by quintessence as well as charged BTZ-like magnetic black hole. First of all, by using Hamilton–Jacobi ansatz and WKB approximation to the field equation of massive vector particles, we get the required tunneling rate of emitted particles and obtain the corresponding Hawking temperature T_h for the black hole (BH) surrounded by quintessence. In order to study the quantum gravity effects, we utilize the generalized Proca and Klein–Gordan equations incorporating the generalized uncertainty principle (GUP) for these BHs and recover their modified tunneling probability as well as accompanying quantum corrected temperatures T'_h.

Keywords: Hawking radiation; quantum tunneling; quantum corrections; Proca equation; modified Klein–Gordan equation.

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1. Introduction

The study of quantum mechanical effects within the foundation of classical general relativity provides numerous interesting phenomena, which play a very important part to understand the quantum gravity theories. In quantum theory, the black

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hole (BH) evaporation as a result of Hawking radiation[1] is one of the important phenomena. During the study of radiation process, the researchers try to combine the gravitation within BH thermodynamics and the quantum mechanics.[2,3] In order to study the Hawking radiation phenomenon, many techniques have been proposed in the literature. Many researchers have studied these radiations for the well-known BHs.[4–42] These radiations can be investigated by the semiclassical approach, depending upon the quantum tunneling method of a particle through the outer horizon of BH from inside to outside.[43,44] The tunneling approach is based on two different techniques, null geodesic[45–47] and Hamilton Jacobi method.[48] These techniques derive the imaginary part of the classical action (I) from the outer horizon by following the classically forbidden trajectory of a particle. The tunneling probability from a BH in both techniques can be defined as

$$\Gamma_{\mathcal{P}} = e^{-\frac{2}{\hbar}\operatorname{Im} I}. \tag{1}$$

This probability formula provides the Hawking temperature of the BH. The presence of a minimal observable length is a common factor in different quantum gravity theories, e.g. loop quantum gravity, noncommutative geometry and string theory.[49–52] The generalized uncertainty principle (GUP) is a straightforward way for understanding this minimal length.[53] The generalized commutation relation can be given as $[x_u, \mathfrak{p}_v] = i\hbar\delta_{uv}[1 + \beta\mathfrak{p}^2]$, where x_u is the modified position operator, while $\mathfrak{p}_v$ stands for modified momentum operator.

The GUP relation can be expressed in the following form:

$$\Delta x \Delta\mathfrak{p} \geq \frac{\hbar}{2}\big[1 + \beta(\Delta\mathfrak{p}^2)\big], \tag{2}$$

where $\beta = \beta_0/\tilde{M}_{\mathfrak{p}^2}$. β_0 and $\tilde{M}_{\mathfrak{p}}$ are the dimensionless parameter and Planck mass, respectively. The GUP relations play a vital role in understanding the nature of BHs, while the quantum effects may be considered as vital effects around the event horizon of a BH. By utilizing the quantum tunneling method, the quantum effects incorporating GUP relations with thermodynamical properties for different spins of BHs have been widely studied.[54–84] Later cosmic observations emphatically suggest that our universe is right now experiencing a stage of accelerated expansion,[85] likely determined by some exotic form known as dark energy. In spite of the mounting observational proof, the nature and cause of dark energy are still like puzzles and have become a matter of vivid discussion. Quintessence is one of the logical possibilities for dark energy with negative pressure.[86] It is evolving, dynamic and spatially heterogeneous element. By comparison within the present context, it would be the fifth dynamical element that has affected the expansion of the universe, additionally to the already known photons, baryons, dark matter and leptons. It can be described by its equation of state, i.e. ($\omega_q = p/\rho$), where ρ and p denote the energy density and pressure for quintessential field, respectively. Moreover, most of the models have the range $0 \geq \omega_q > -1$, though a cosmological consistent has $\omega_q = -1$. The accelerating expansion depends upon the value of ω_q, as smaller value gives a more accelerating effect. The Einstein field equations for four- and d-dimensional BHs with quintessential field have been studied.[87,88]

The investigation of $(2 + 1)$-dimensional BHs gives an important knowledge in comprehension of low-dimensional gravity theories and their quantum partners. The Banados–Teitelboim–Zanelli (BTZ) BH is such a model in $(2+1)$-dimensional gravity.[89] This BH is surprisingly comparative to $(3+1)$-dimensional BH. Just like the Kerr BH, it has an outer horizon and an inner horizon. It may be completely portrayed by charge, mass and angular momentum. Additionally, it has thermodynamical properties closely resembling the $(3+1)$-dimensional BH. The BTZ solution has been additionally discussed in the domain of $(2+1)$-dimensional quantum gravity. Moreover, it will be very interesting to study the BTZ-like BH with magnetic field. It is a well-known fact that the electric field is related to the temporal component of gauge potential A_t, whereas the electric field is linked with the angular part of the gauge, potential A_φ. These types of BHs, only magnetic gauge can be used to derive BH solutions. It is accepted that the BHs in $2 + 1$ dimensions will generally provide a basic laboratory and an outstanding comprehension for examining the general perspectives of BH physics. The other main reason and motivation to choose $(2+1)$-dimensional BHs is the presence of AdS/CFT correspondence[90] that relates the thermodynamical properties of BHs with AdS/CFT duality. In recent years, the researchers have paid much attention to analyze the thermal properties of $(2 + 1)$-dimensional BHs, particularly with a spacetime having non-vanishing cosmological constant. The main purpose of this paper is to investigate the Hawking radiation phenomenon with GUP effects under quintessential and magnetic field from $(2 + 1)$-dimensional BHs. For this purpose, first, we consider a $(2 + 1)$-dimensional BH with quintessential field. Then by utilizing the Hamilton–Jacobi technique, we calculated the tunneling probability and Hawking temperature for spin-1 particles. Moreover, we have analyzed the corrected Hawking temperature for spin-1 and spin-0 particles. Further, we consider a magnetically charged version of BTZ-like BH. We have studied the modified tunneling probability and corrected Hawking temperature for spin-1 and spin-0 particles for BTZ-like BH with magnetic field. The paper is outlined as follows. Section 2 contains the information about metric of $(2 + 1)$-dimensional BH with quintessential field. Section 3 is devoted for studying the tunneling probability and Hawking temperature for quintessential BH for vector particles. In Sec. 4, we investigate the modified tunneling probability and the corresponding temperature for vector particles from quintessential BH under GUP effects. Section 5 is based on the quantum gravity effects for vector particles from BTZ-like magnetic BH. In Sec. 6, we investigate the quantum corrections for scalar particles from quintessential BH. Section 7 represents the modified tunneling rate and related effective Hawking temperature for spin-0 particles from BTZ-like BH with magnetic field. In Sec. 8, we summarize the results of our work.

2. Metric for (2 + 1)-Dimensional Black Hole with Quintessence

The line element for three-dimensional BH surrounded by quintessence is defined as[54]

$$ds^2 = -\frac{r^2}{l^2} g(r) dt^2 + \frac{l^2}{r^2} g(r)^{-1}\, dr^2 + r^2\, dx^2, \tag{3}$$

where

$$g(r) = 1 - \left(\frac{r_+}{r}\right)^{\rho}, \quad \rho = 2(1 + \omega_q),$$

and $r_+ = (Ml^2)^{1/\rho}$ represents the event horizon, M specifies the mass of BH, ω_q shows the quintessence parameter with range $-1 < \omega_q < -\frac{1}{3}$, l stands for AdS radius which belongs to cosmological constant by $l^2 = -1/\Lambda$, moreover r and x are the radial and planar coordinates with ranges $(0 < r < \infty)$ and $(-\infty < x < \infty)$, respectively.

The line element (3) can also be rewritten as

$$ds^2 = -X(r) dt^2 + \frac{1}{Y(r)} dr^2 + Z(r) dx^2, \tag{4}$$

the above metric functions can be defined as follows:

$$X(r) = \frac{r^2}{l^2}\left[1 - \left(\frac{r_+}{r}\right)^{\rho}\right], \quad Y(r) = \frac{l^2}{r^2}\left[1 - \left(\frac{r_+}{r}\right)^{\rho}\right]^{-1}, \quad Z(r) = r^2.$$

It is important to mention here that for $\omega_q = 0$, the line element reduces into BTZ BH metric.[55]

3. Vector Particles Tunneling

This section is devoted to the study of the tunneling spectrum for spin-1 particles at event horizon of BH by using Proca equation under vector field Ψ_v. The Proca equation can be expressed as[24]

$$\frac{1}{\sqrt{-\tilde{g}}} \partial_u \left(\sqrt{-\tilde{g}} \Psi^{vu}\right) + \frac{m^2}{\hbar^2} \Psi^v = 0 \quad (u, v = 0, 1, 2), \tag{5}$$

where $\tilde{g}$ denotes the determinant of the metric and m is mass of emitted particle. Also, Ψ^{uv} represents anti-symmetric tensor which can be defined as

$$\Psi_{uv} = \mathcal{D}_u \Psi_v - \mathcal{D}_v \Psi_u = \partial_u \Psi_v - \partial_v \Psi_u, \tag{6}$$

where $\mathcal{D}_u$ stands for covariant derivative. We will get the similar results for W^+ and W^- bosons, the equation of motion of tunneling process for both particles is same. But, here we only deal with W^+ boson particles.

The wave function for spin-1 particles by considering the WKB approximation is defined as[56]

$$\Psi_v = (a_0, a_1, a_2) \exp\left[\frac{i}{\hbar} I(t, r, x)\right], \tag{7}$$

where a_0, a_1 and a_2 are arbitrary constants and the action $I(t,r,x)$ is given as

$$I(t,r,x) = I_0(t,r,x) + \hbar I_1(t,r,x) + \hbar^2 I_2(t,r,x) + \cdots . \tag{8}$$

After substituting the covariant components $\psi_0 = -X\psi^0$, $\psi_1 = (1/Y)\psi^1$, $\psi_2 = Z\psi^2$, the metric (4) and Eq. (7) into Eq. (5) neglecting the higher terms of $\hbar$, we obtain the following set of equations:

$$Y[a_0(\partial_r I)^2 - a_1(\partial_t I)(\partial_r I)] + \frac{1}{Z}[a_0(\partial_x I)^2 - a_2(\partial_t I)(\partial_x I)] + m^2 a_0 = 0, \tag{9}$$

$$\frac{1}{X}[a_0(\partial_t I)(\partial_r I) - a_1(\partial_r I)^2] + \frac{1}{Z}[a_1(\partial_x I)^2 - a_2(\partial_r I)(\partial_x I)] + m^2 a_1 = 0, \tag{10}$$

$$\frac{1}{X}[a_0(\partial_t I)(\partial_x I) - a_2(\partial_t I)^2] + Y[a_2(\partial_r I)^2 - a_1(\partial_x I)(\partial_r I)] + m^2 a_2 = 0. \tag{11}$$

We consider a particle's action in the following form:

$$I = -Et + R(r) + jx + C, \tag{12}$$

where E denotes the energy of the emitted particles and C represents the complex constant. The above set of equations gives a 3×3 matrix equation, which can be given as[56]

$$\aleph(a_0, a_1, a_2)^T = 0, \tag{13}$$

where

$$\aleph = \begin{pmatrix} Y(\partial_r R)^2 + \dfrac{(\partial_x j)^2}{Z} + m^2 & YE\partial_r R & \dfrac{E\partial_x j}{Z} \\ \dfrac{E\partial_r R}{X} & \dfrac{E^2}{X} - \dfrac{(\partial_x j)^2}{Z} - m^2 & \dfrac{(\partial_r R)\partial_x j}{Z} \\ \dfrac{E\partial_x j}{X} & Y(\partial_r R)\partial_x j & \dfrac{E^2}{X} - B(\partial_r R)^2 - m^2 \end{pmatrix}. \tag{14}$$

In order to obtain the nontrivial solution, we set $\det \aleph = 0$ and obtain

$$R_\pm(r) = \pm\sqrt{\frac{E^2 - X(r)\left(m^2 + \frac{(\partial_x j)^2}{Z(r)}\right)}{X(r)Y(r)}} dr. \tag{15}$$

The R_+ and R_- represent the outgoing motion (with momentum $\partial_r I > 0$) and ingoing motion (with momentum $\partial_r I < 0$), respectively.

The $R_\pm$ possesses a simple pole at $r = r_+$ and it gives the same imaginary part for both outgoing/ingoing solutions.

$$R_\pm(r) = \pm\frac{2\pi i E l^2}{\rho^2 r_+}, \tag{16}$$

$$\mathrm{Im}\, R_\pm(r) = \pm\frac{2\pi E l^2}{\rho^2 r_+}. \tag{17}$$

The evaporation process in BHs is a quantum process[1] and the absorption/emission probabilities for crossing the event horizon in and out can be defined as[57]

$$\mathcal{P}[\text{emission}] = \mathcal{P}_+ = -\exp\left[-\frac{2}{\hbar}(\operatorname{Im} R_+ + \operatorname{Im} C)\right], \tag{18}$$

$$\mathcal{P}[\text{absorption}] = \mathcal{P}_- = \exp\left[-\frac{2}{\hbar}(\operatorname{Im} R_+ + \operatorname{Im} C)\right]. \tag{19}$$

It is important to mention here that a factor of two problem arises because according to classical general theory of relativity, a particle can be absorbed in the BH, while the reverse process is forbidden. In this regard, ingoing classical trajectory exists, while the outgoing classical trajectory is forbidden. Hence, use of the classical condition for outgoing particle is meaningless and to solve this problem, we choose the ingoing probability as ($\mathcal{P}_- \simeq e^{-\frac{2}{\hbar}\operatorname{Im} R_-} = 1$). Due to the fact that $R_+ = -R_-$, we have $\operatorname{Im} C = \operatorname{Im} R_+$ and therefore from Eq. (18), we get

$$\mathcal{P}[\text{emission}] = \exp\left[-\frac{4}{\hbar}(\operatorname{Im} R_+)\right], \tag{20}$$

So, the total tunneling probability for radiated particles from inside to outside ($\mathcal{P}_- = 1$) can be defined as follows:

$$\Gamma_{\mathcal{P}} = \frac{\mathcal{P}[\text{emission}]}{\mathcal{P}[\text{absorption}]} \simeq \exp\left[-\frac{4}{\hbar}(\operatorname{Im} R_+)\right], \tag{21}$$

$$\Gamma_{\mathcal{P}} = \exp\left[-\frac{8\pi l^2 E}{\hbar\rho^2 r_+}\right]. \tag{22}$$

The surface gravity κ for the given BH can be calculated as

$$\kappa = \left[\frac{\rho^2 r_+}{4\pi l^2}\right]. \tag{23}$$

In order to obtain the corresponding Hawking temperature, we use Boltzmann formula $\Gamma_{\mathcal{P}} = e^{-E/T_h}$ and get

$$T_h = \frac{\kappa}{2\pi} = \left[\frac{\rho^2 r_+}{8\pi l^2}\right] \quad (\hbar = 1). \tag{24}$$

This is the required Hawking temperature. It depends upon the curvature radius l and quintessence parameter ω_q (i.e. $\rho = 2(1+\omega_q)$. For the different values of ω_q, we can derive the Hawking temperature in the following form:

when $\omega_q = -1/3$

$$T_h = \left[\frac{2r_+}{9\pi l^2}\right], \tag{25}$$

when $\omega_q = -1/2$

$$T_h = \left[\frac{r_+}{18\pi l^2}\right], \tag{26}$$

when $\omega_q = -2/3$

$$T_h = \left[\frac{r_+}{8\pi l^2}\right]. \tag{27}$$

4. Quantum Gravity Effects for Spin-1 Particles

In this section, we study the corrected tunneling rate for spin-1 particles through the horizon, we choose a Lagrangian equation under quantum gravity effects. The GUP modified Lagrangian equation for spin-1 particles with vector field Ψ_u can be given as[58]

$$\pounds^{\mathrm{GUP}} = -\frac{1}{2}[\mathfrak{D}_u\Psi_v - \mathfrak{D}_v\Psi_u][\mathfrak{D}^u\Psi^v - \mathfrak{D}^v\Psi^u] - \frac{m^2}{\hbar^2}\Psi^v\Psi_u. \tag{28}$$

The modified equation of motion for bosons particles can be defined as follows[58]:

$$\partial_u\big(\sqrt{-\tilde{g}}\Psi^{uv}\big) + \hbar^2\beta\partial_0\partial_0\partial_0\big(\sqrt{-\tilde{g}}g^{00}\Psi^{0v}\big) - \hbar^2\beta\partial_i\partial_i\partial_i\big(\sqrt{-\tilde{g}}g^{ii}\Psi^{iv}\big) - \sqrt{-\tilde{g}}\frac{m^2}{\hbar^2}\Psi^v = 0. \tag{29}$$

Here, m shows the mass of emitted particle. The generalized anti-symmetric tensor Ψ_{uv} can be defined as[58]

$$\Psi_{uv} = (1-\beta\hbar^2\partial_u^2)\partial_u\Psi_v - (1-\beta\hbar^2\partial_v^2)\partial_v\Psi_u. \tag{30}$$

The modified tensor Ψ_{iu}, $i = 1, 2$ stands for spatial components and Ψ_{0u}, 0 states the time coordinate. Also, the correction parameter β can be given as $\beta = \beta_0/\tilde{M}_{\mathfrak{p}^2}$, where $\tilde{M}_{\mathfrak{p}^2}$ denotes the minimal length.[64]

The wave function for emitted particles with vector field Ψ^u can be expressed as[56]

$$\Psi_v = (a_0, a_1, a_2)\exp\left[\frac{i}{\hbar}I(t,r,x)\right], \tag{31}$$

By using the metric components from Eq. (4) and values from Eqs. (30) and (31) into Eq. (29), we get a set of equations

$$Y[a_0(\partial_r I)^2\tilde{P}_1^2 - a_1(\partial_r I)(\partial_t I)\tilde{P}_1\tilde{P}_0] + \frac{1}{Z}[a_0(\partial_x I)^2\tilde{P}_2^2 - a_2(\partial_x I)(\partial_t I)\tilde{P}_2\tilde{P}_0] + a_0 m^2 = 0, \tag{32}$$

$$-\frac{1}{X}[a_1(\partial_t I)^2\bar{P}_0^2 - a_0(\partial_t I)(\partial_r I)\tilde{P}_0\tilde{P}_1] + \frac{1}{Z}[a_1(\partial_x I)^2\tilde{P}_2^2 - a_2(\partial_x I)(\partial_r I)\tilde{P}_2\tilde{P}_1] + a_1 m^2 = 0, \tag{33}$$

$$-\frac{1}{X}[a_2(\partial_t I)^2\tilde{P}_0^2 - a_0(\partial_t I)(\partial_x I)\tilde{P}_0\tilde{P}_2] + Y[a_2(\partial_r I)^2\tilde{P}_1^2 - a_1(\partial_r I)(\partial_x I)\tilde{P}_1\tilde{P}_2] + a_2 m^2 = 0, \tag{34}$$

where

$$\tilde{P}_0 = 1 + \beta \frac{1}{X(r)}(\partial_t I)^2, \quad \tilde{P}_1 = 1 + \beta Y(r)(\partial_r I)^2,$$
$$\tilde{P}_2 = 1 + \beta \frac{1}{Z(r)}(\partial_x I)^2. \tag{35}$$

By assuming the separation of variables technique

$$I = -Et + R(r) + jx + C, \tag{36}$$

where E denotes the energy of the emitted particles and C represents the complex constant. After inserting Eq. (36) in the above set of equations, a matrix equation can be given as[56]

$$\aleph(a_0, a_1, a_2)^T = 0, \tag{37}$$

where

$$\aleph = \begin{pmatrix} YR'^2\tilde{P}_1^2 + \frac{j^2}{Z}\tilde{P}_2^2 + m^2 & YER'\tilde{P}_0\tilde{P}_1 & \frac{Ej\tilde{P}_0\tilde{P}_2}{Z} \\ \frac{-ER'\tilde{P}_0\tilde{P}_1}{X} & \frac{-E^2\tilde{P}_0^2}{X} + \frac{j^2\tilde{P}_2^2}{Z} + m^2 & \frac{-jR'\tilde{P}_1\tilde{P}_2}{Z} \\ \frac{-Ej\tilde{P}_0\tilde{P}_2}{X} & -YjR'\tilde{P}_1\tilde{P}_2 & \frac{-E^2\tilde{P}_0^2}{X} + YR'^2\tilde{P}_1^2 + m^2 \end{pmatrix}, \tag{38}$$

where $R' = \partial_r R$. When we set $\det \aleph = 0$ from Eq. (38), we obtain a nontrivial solution in the form

$$\partial_r R_{\pm} = \pm\sqrt{\frac{-m^2}{Y(r)} + \frac{E^2}{Y(r)X(r)} - \frac{j^2}{Y(r)Z(r)}}\left(1 + \frac{\mathfrak{F}_1}{\mathfrak{F}_2}\beta\right), \tag{39}$$

where

$$\mathfrak{F}_1 = -3m^4X(r)Z(r) + 6E^2m^2Z(r) - 6m^2j^2X(r) - \frac{6j^4X(r)}{Z(r)} + 6E^2j^2$$
$$- \frac{7X(r)j^2}{Z(r)} - \frac{3X(r)j^2}{2m^2Z(r)^2} + \frac{3X(r)j^2}{2m^2Z(r)^2},$$
$$\mathfrak{F}_2 = -m^2X(r)Z(r) + E^2Z(r) - j^2X(r).$$

After integrating Eq. (39), we calculate the imaginary part of the radial action in the following form:

$$\operatorname{Im} R_{\pm} = \pm\iota\pi\frac{E}{X'(r)}(1 + \beta\chi), \tag{40}$$

where $\chi > 0$ and can be given as

$$\chi = 6m^2 + \frac{6j^2}{r_+^2}.$$

Here, R_+/R_- stand for radial functions for the outgoing/incoming particles, respectively. The tunneling rate for spin-1 particles at event horizon can be derived as

$$\Gamma_{\mathcal{P}} = \frac{\mathcal{P}[\text{emission}]}{\mathcal{P}[\text{absorption}]} \simeq \exp\left[-\frac{4}{\hbar}(\operatorname{Im} R_+)\right], \tag{41}$$

$$\Gamma_{\mathcal{P}} = \exp\left[-\frac{8\pi l^2 E}{\hbar \rho^2 r_+}(1+\beta\chi)\right]. \tag{42}$$

The corresponding effective Hawking temperature can be calculated as follows:

$$T'_h = \frac{\rho^2 r_+}{8\pi l^2(1+\beta\chi)} = T_h(1-\beta\chi), \tag{43}$$

where

$$T_h = \frac{\rho^2 r_+}{8\pi l^2},$$

this is the original Hawking temperature of the given BH. Equation (43) represents the effective Hawking temperature under quantum gravity effects.

In case of quantum corrections, we can observe the stopping of BH evaporation. Quantum corrections decelerate the increase in Hawking temperature during the radiation phenomenon. During the evaporation process, we can achieve a balance state, where the evaporation stops and remnants are left. When ($\beta = 0$), we can get the original Hawking temperature T_h for a given BH surrounded by the quintessence. For different values of quintessence, we can obtain the corrected Hawking temperature as follows:

when $\omega_q = -1/3$

$$T'_h = \left[\frac{2r_+}{9\pi l^2}(1-\beta\chi)\right], \tag{44}$$

when $\omega_q = -1/2$

$$T'_h = \left[\frac{r_+}{18\pi l^2}(1-\beta\chi)\right], \tag{45}$$

when $\omega_q = -2/3$

$$T'_h = \left[\frac{r_+}{8\pi l^2}(1-\beta\chi)\right]. \tag{46}$$

5. Quantum Gravity Effects for Charged BTZ-Like Magnetic Black Hole

The line element for $(2+1)$-dimensional BTZ-like BH with magnetic charge q_m can be defined as[9]

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{1}{G(r)}dr^2 + l^2 G(r)\tau^2\, d\varphi^2, \tag{47}$$

where $G(r)$ is the metric function, l denotes the radius curvature with negative cosmological constant $\Lambda = -\frac{1}{l^2}$, r represents the radial coordinate and

$$G(r) = \frac{r^2}{l^2} - \left[\tilde{M} + 8q_m^2 l^2 \tau^2 \ln\left(\frac{r}{l}\right)\right], \tag{48}$$

where $\tilde{M}$ shows the mass of the BH and τ is the arbitrary parameter. Equation (47) can be rewritten in the following form:

$$ds^2 = \tilde{X}(r)dt^2 + \tilde{Y}(r)dr^2 + \tilde{Z}(r)d\varphi^2, \tag{49}$$

where the metric functions $\tilde{X}(r)$, $\tilde{Y}(r)$ and $\tilde{Z}(r)$ can be given as

$$\tilde{X}(r) = -\frac{r^2}{l^2}, \quad \tilde{Y}(r) = \frac{1}{G(r)}, \quad \tilde{Z}(r) = l^2 G(r)\tau^2. \tag{50}$$

The gauge potential of BH is given as follows:

$$A_\mu = -2q_m l^2 \tau^2 k(r)\delta_\mu^\varphi, \tag{51}$$

where A_φ is the only nonzero component due to magnetic field and $k(r)$ represents the arbitrary function of r.

In order to investigate the corrected tunneling rate and Hawking temperature for BTZ-like charged magnetic BH, we consider the following modified wave equation for massive bosons as[11]

$$\begin{aligned}\partial_u\left(\sqrt{-\tilde{g}}\Psi^{vu}\right) &+ \sqrt{-\tilde{g}}\frac{m^2}{\hbar^2}\Psi^v + \sqrt{-\tilde{g}}\frac{\iota}{\hbar}eA_u\Psi^{vu} + \sqrt{-\tilde{g}}\frac{\iota}{\hbar}e\tilde{F}^{vu}\Psi_u \\ &+ \hbar^2\beta\partial_0\partial_0\partial_0\left(\sqrt{-\tilde{g}}g^{00}\Psi^{0v}\right) - \hbar^2\beta\partial_i\partial_i\partial_i\left(\sqrt{-\tilde{g}}g^{ii}\Psi^{iv}\right) = 0.\end{aligned} \tag{52}$$

The anti-symmetric tensor for the charged particles can be defined as[11]

$$\begin{aligned}\Psi_{uv} &= (1-\beta\hbar^2\partial_u^2)\partial_u\Psi_v - (1-\beta\hbar^2\partial_v^2)\partial_v\Psi_u \\ &\quad + (1-\beta\hbar^2\partial_u^2)\frac{\iota}{\hbar}eA_u\Psi_v - (1-\beta\hbar^2\partial_v^2)\frac{\iota}{\hbar}eA_v\Psi_u, \quad \text{and} \\ \tilde{F}_{uv} &= \nabla_u A_v - \nabla_v A_u, \quad \text{where } \tilde{\nabla}_0 = (1+\hbar^2\beta g^{00}\nabla_0^2)\nabla_0, \\ &\quad \tilde{\nabla}_i = (1-\hbar^2\beta g^{ii}\nabla_i^2)\nabla_i.\end{aligned} \tag{53}$$

By using metric (49) into Eq. (52) and following the same procedure of Sec. 4, we obtain the imaginary part for BTZ-like magnetic BH as

$$\operatorname{Im} R_\pm = \pm\iota\pi\frac{(E - j\omega - eA_\phi)l^2 r_+}{r_+^2 - 4q_m^2\tau^2 l^4}, \tag{54}$$

where R_+ and R_- represent radial function for the outgoing and incoming particles, respectively. For our standard coordinates of the BH metric, the tunneling of outgoing particles can be obtained by taking an infinitesimal half circle below the pole $r = r_+$, while for the incoming particle, such a contour is taken above the pole. Further, in order to calculate the semi-classical tunneling probability, it is required that the resulting wave equation must be multiplied by its complex conjugate. In this way, the part of the trajectory starts from outside of the BH and continues to the observer which will not contribute to the calculation of the final tunneling probability and can be ignored because it will be completely real. Therefore, the only part of the trajectory that contributes to the tunneling probability is the contour around the BH horizon. We set the probability of incoming spin-1 particles as 100% by setting ($\mathcal{P}_- \simeq e^{-\frac{2}{\hbar}\operatorname{Im} R_-} = 1$) to solve the factor of two problems. Due to the fact that $R_+ = -R_-$, we have $\operatorname{Im} C = \operatorname{Im} R_+$ and therefore from Eq. (54), we get

$$\Gamma_{\mathcal{P}} = \exp\left[-2\pi \frac{l^2(E - j\Omega_h - eA_\varphi)}{r_+^2 - 4q_m^2\tau^2 l^4}(1 + \beta\chi)\right] \quad (\text{for } \hbar = 1). \tag{55}$$

The modified tunneling probability of vector particles depends upon the energy E, the charged potential of BH A_φ, the magnetic charge q_m, particle's charge e, curvature radius ℓ, correction parameter β, angular momentum j and angular velocity Ω_h.

After utilizing Boltzmann factor $\Gamma_{\mathcal{P}} = \exp[(E - j\Omega_h - eA_\varphi)/T_h]$, the corresponding effective temperature for given BH can be calculated as follows:

$$T_h' = \frac{r_+^2 - 4q_m^2\tau^2 l^4}{2\pi l^2 r_+(1 + \beta\chi)} = T_h(1 - \beta\chi), \tag{56}$$

where

$$T_h = \frac{r_+^2 - 4q_m^2\tau^2 l^4}{2\pi l^2 r_+}. \tag{57}$$

T_h is the original Hawking temperature of BTZ-like magnetic BH. If we consider $\beta = 0$, then the modified temperature T_h' convert into original Hawking temperature T_h of BTZ-like magnetic BH.[9] The effective Hawking temperature depends upon magnetic charge q_m of BH, the curvature radius and correction parameter β.

6. Quantum Corrections of Spin-0 Particles for Quintessential BH

This section is devoted to the study of quantum gravity effects on the tunneling of massive scalar particles from a BH with quintessential field.

We investigate the tunneling process of spin-0 particles for given BH and calculate the modified tunneling probability and corresponding corrected Hawking temperature. For this purpose, we consider the modified Klein–Gordon equation with scalar field ϕ as[63]

$$-(i\hbar)^2\partial_t\partial^t\phi = [m^2 + (-i\hbar)^2\partial_i\partial^i][1 - 2\beta(m^2 + (-i\hbar)^2\partial_i\partial^i)]\phi, \tag{58}$$

The generalized Klein–Gordan equation in the background of quintessential BH metric (4), obtains the form[64]

$$\frac{\hbar^2}{X(r)}\frac{\partial^2\phi}{\partial t^2}-\frac{\hbar^2}{Z(r)}\frac{\partial^2\phi}{\partial x^2}-2\hbar^4\beta Y(r)\frac{\partial^2\phi}{\partial r^2}\left[-Y(r)\frac{\partial^2\phi}{\partial r^2}\right]$$
$$-2\frac{\hbar^4\beta}{Z(r)}\frac{\partial^2\phi}{\partial x^2}\left[-\frac{1}{Z(r)}\frac{\partial^2\phi}{\partial x^2}\right]-\hbar^2 Y(r)\frac{\partial^2\phi}{\partial r^2}+m^2(1-2\beta m^2)\phi=0. \tag{59}$$

The wave function for spin-0 particles can be defined as follows:

$$\phi(t,r,x)=\exp\left[\frac{i}{\hbar}I(t,r,x)\right]. \tag{60}$$

By inserting Eq. (60) into Eq. (59), only for first-order term in $\hbar$, we obtain

$$\frac{1}{X(r)}\left(\frac{\partial I}{\partial t}\right)^2=Y(r)\left(\frac{\partial I}{\partial r}\right)^2+\frac{1}{Z(r)}\left(\frac{\partial I}{\partial x}\right)^2+m^2+\frac{\beta}{Z(r)^2}\left(\frac{\partial I}{\partial x}\right)^4$$
$$-\beta\left[m^4-2Y(r)^2\left(\frac{\partial I}{\partial r}\right)^4\right]. \tag{61}$$

In order to solve this equation, we consider the following particle's action:

$$I(t,r,x)=-Et+jx+R(r). \tag{62}$$

Here, $R(r)=R_0(r)+\beta R_1(r)$,[59] so the radial integral $R(r)$ gets the form

$$R_{\pm}(r)=\sqrt{\frac{E^2-X(r)\left(m^2+\frac{j^2}{Z(r)}\right)}{Y(r)}}(1+\beta\Im). \tag{63}$$

After solving the above integral, we obtain the following relation:

$$R_{\pm}(r)=\pm i\pi\frac{E}{X'(r)}(1+\beta\Im), \tag{64}$$

where $\Im>0$ can be given as

$$\Im=\left(\frac{X(r)\left(m^2+\frac{j^4}{Z(r)^2}\right)}{E^2-X(r)\left(m^2+\frac{j^2}{Z(r)}\right)}-\frac{E^2-X(r)\left(m^2+\frac{j^2}{Z(r)}\right)}{Y(r)}\right). \tag{65}$$

Here, R_+ and R_- are the radial functions for the outgoing/incoming particles, respectively. The tunneling probability for spin-0 particles at horizon can be given as follows:

$$\Gamma_{\mathcal{P}}=\frac{\mathcal{P}[\text{emission}]}{\mathcal{P}[\text{absorption}]}\simeq\exp\left[-\frac{4}{\hbar}(\operatorname{Im}R_+)\right], \tag{66}$$

$$\Gamma_{\mathcal{P}}=\exp\left[-\frac{8\pi l^2 E}{\hbar\rho^2 r_+}(1+\beta\Im)\right]. \tag{67}$$

The corresponding corrected Hawking temperature for scalar particles can be calculated as follows:

$$T'_h = \frac{\rho^2 r_+}{8\pi l^2(1+\beta\Im)} = T_h(1-\beta\Im), \tag{68}$$

where

$$T_h = \frac{\rho^2 r_+}{8\pi l^2},$$

this is the original Hawking temperature of the given BH. Equation (68) represents the effective Hawking temperature under quantum gravity effects. For different values of quintessence, we can obtain the corrected Hawking temperature as follows:

when $\omega_q = -1/3$

$$T'_h = \left[\frac{2r_+}{9\pi l^2}(1-\beta\Im)\right], \tag{69}$$

when $\omega_q = -1/2$

$$T'_h = \left[\frac{r_+}{18\pi l^2}(1-\beta\Im)\right], \tag{70}$$

when $\omega_q = -2/3$

$$T'_h = \left[\frac{r_+}{8\pi l^2}(1-\beta\Im)\right]. \tag{71}$$

7. Quantum Corrections for BTZ-Like Magnetic BH

In this section, we calculate the modified tunneling rate and corresponding effective Hawking temperature for BTZ-like BH in the presence of magnetic field. For this purpose, we use the modified Klein–Gordan equation with scalar field ϕ

$$-(i\hbar)^2\partial_t\partial^t\phi = [m^2 + (-i\hbar)^2\partial_i\partial^i][1 - 2\beta(m^2 + (-i\hbar)^2\partial_i\partial^i)]\phi. \tag{72}$$

By using the metric Eq. (47) in Eq. (72) and following the same procedure of Sec. 6, we derive the modified tunneling probability for the given BH in the following form:

$$\Gamma_{\mathcal{P}} = \exp\left[-2\pi\frac{l^2 E}{r_+^2 - 4q_m^2\tau^2 l^4}(1+\beta\Im)\right], \tag{73}$$

where

$$\Im = \left(\frac{X(r)\left(m^2 + \frac{j^4}{Z(r)^2}\right)}{E^2 - X(r)\left(m^2 + \frac{j^2}{Z(r)}\right)} - \frac{E^2 - X(r)\left(m^2 + \frac{j^2}{Z(r)}\right)}{Y(r)}\right). \tag{74}$$

The corresponding corrected Hawking temperature at BH horizon for spin-0 particles can be deduced in the form

$$T'_h = \frac{r_+^2 - 4q_m^2\tau^2 l^4}{2\pi l^2 r_+(1+\beta\Im)} = T_h(1-\beta\Im), \tag{75}$$

where $T_h = \frac{r_+^2 - 4q_m^2 \tau^2 l^4}{2\pi l^2 r_+}$ shows the standard Hawking temperature for BTZ-like magnetic BH. When we neglect quantum gravity effect, i.e. ($\beta = 0$), then the corrected temperature reduced into standard Hawking temperature of BTZ-like BH.

8. Conclusion

In this paper, we have investigated the Hawking radiation process via quantum tunneling phenomenon for vector particles from $(2+1)$-dimensional BH with quintessence. In this regard, we have considered the Hamilton–Jacobi technique and WKB method to the field equation of massive vector particles. We have calculated the required tunneling rate of emitted particles and their corresponding Hawking temperature, T_h. Moreover, we have studied the quantum gravity effect on the tunneled spin-1 (vector) and spin-0 (scalar) particles from $(2+1)$-dimensional BHs in the background of quintessential and BTZ-like magnetic BHs. First, we consider the modified Proca and Klein–Gordon equations that describe the motion of spin-1 and spin-0 particles, respectively. Then, by utilizing the Hamilton–Jacobi method, the tunneling probabilities and effective Hawking temperatures T_h' for both type of particles from the given BHs are calculated. We note that the Hawking temperature and corrected Hawking temperature are not only based on the BHs properties, but also depend on the radiated particles energy, mass, and angular momentum.

It is worth mentioning that the corrected temperatures obtained in Eqs. (43) and (68) for quintessential BH and in Eqs. (56) and (75) for BTZ-like magnetic BH look identical for spin-1 and spin-0 particles. Moreover, we conclude that also the Hawking temperature is independent for types of emitted particles. But the relation to angular momentum and mass is not entirely identical in these results. These temperatures also look preserved over charge and energy. From our analysis, we have concluded that the Hawking temperature at which particles tunnel through the horizon is independent of the species of particles. In particular, the nature of the background BH geometries, for the particles having different spins (either spin up or down) or zero spin, the tunneling probabilities will be seen to be the same by considering semi-classical effects. Thus, their corresponding Hawking temperatures must be the same for all kinds of particles.

The corrected temperature for quintessential BH depends upon the curvature radius l, correction parameter β and quintessence parameter ω_q (i.e. $\rho = 2(1+\omega_q)$). When we neglect the quantum gravity effects ($\beta = 0$), then the modified Hawking temperature of quintessential BH is reduced into standard Hawking temperature of Eq. (24). Similarly, the corrected Hawking temperature of BTZ-like magnetic is BH reduced into standard Hawking temperature of Ref. 9. Moreover, the effective Hawking temperature for BTZ-like magnetic BH depends upon magnetic charge q_m of BH, the curvature radius and correction parameter β. When the magnetic charge q_m of BH increases, the Hawking temperature decreases in Eq. (75), on the other hand, when the quintessence parameter ω_q increases, the Hawking temperature also increases in Eq. (24).

It is important to mention here that the quantum corrections slow down the increase in Hawking temperature during evaporation process. At the point when the mass of BH approaches to its minimum value, it stops radiating and BH remnants are left.

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