

Eruptive Massive Vector Particles of 5-Dimensional Kerr-Gödel Spacetime

A. Övgün^{1,2}  · I. Sakalli²

Received: 28 April 2017 / Accepted: 28 September 2017 / Published online: 5 October 2017
© Springer Science+Business Media, LLC 2017

Abstract In this paper, we investigate Hawking radiation of massive spin-1 particles from 5-dimensional Kerr-Gödel spacetime. By applying the WKB approximation and the Hamilton-Jacobi ansatz to the relativistic Proca equation, we obtain the quantum tunneling rate of the massive vector particles. Using the obtained tunneling rate, we show how one impeccably computes the Hawking temperature of the 5-dimensional Kerr-Gödel spacetime.

Keywords Hawking radiation · Vector particles · Quantum tunneling · Kerr-Gödel spacetime

1 Introduction

Today, there are numerous methods that confirm the outstanding discovery of Hawking [1–3], which theoretically showed that a black hole can sparkle as such in a blackbody radiation. In the last decade, semi-classical methods of modeling Hawking radiation as a tunneling process have garnered lots of interest [4–48]. Those semi-classical methods model the Hawking radiation as if a tunneling process, which uses the WKB approximation to compute the imaginary part of the classically unpermitted action (ImS) of the trajectory that crosses the event horizon. By this way, the tunneling probability is computed via the expression of $\Gamma \propto \exp(-2ImS)$.

✉ A. Övgün
ali.ovgun@pucv.cl

I. Sakalli
izzet.sakalli@emu.edu.tr

¹ Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, Chile

² Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Turkey

The studies about the Hawking radiation of massive vector particles have been gained attention as from the publications of Kruglov [49, 50]. His main approach is to apply the Hamilton-Jacobi method to the Proca equation [51]. As is well known, the vector particles (e.g. Z , W^\pm) play important role in the high energy physics and in the standard model [52, 53]. Since the Hawking quanta should be highly energetic tiny particles, so it is plausible to consider the Hawking radiation of the massive vector particles from various geometries of black holes. A reader would appreciate that the most subtle black holes are the rotating ones defined in the higher dimensions. From this viewpoint, we aim to extend the Kruglov’s studies (and his followers’ works [54–65]) to the radiation spectrum of massive vector particles spraying from a 5-dimensional Kerr-Gödel black hole (5DKGBH). Thus, we aim to calculate Hawking temperature [66] of the 5DKGBH.

One of the splendid exact solutions of Einstein equations with a cosmological constant and homogeneous pressure-free matter is the Gödel universe [67]. Gödel spacetime possesses a peculiar feature: there always exist closed timelike or null curves. On the other hand, the generic version of the Gödel spacetime, which is the exact solution of minimal supergravity (preserving some number of supersymmetries) in 5 dimensions was produced in [68, 69]. The consistency of this solution to the string theory was further studied in [70, 71] in which the supersymmetric Gödel universes of [68, 69] are shown to be related with the pp-wave solutions having T -duality [72, 73].

The organization of the paper is as follows. In Section 2, we introduce the 5DKGBH spacetime and demonstrate its thermodynamic features. We then study the quantum tunneling of massive spin-1 particles from the 5DKGBH by employing the Proca equation. In sequel, we compute the quantum tunneling rate and obtain the corresponding temperature of the black hole surface. In Section 3, we summarize our discussions.

2 Kerr-Gödel Black Hole in 5-Dimensions

In Boyer-Lindquist coordinates, the 5DKGBH spacetime is given by [74]

$$ds^2 = -f(r) \left(dt + \frac{a(r)}{f(r)} \sigma_3 \right)^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4} (\sigma_1^2 + \sigma_2^2) + \frac{r^2 V(r)}{4f(r)} \sigma_3^2, \tag{1}$$

where

$$f(r) = 1 - \frac{2m}{r^2}, \tag{2}$$

$$a(r) = jr^2 + \frac{ml}{r^2}, \tag{3}$$

$$V(r) = 1 - \frac{2m}{r^2} + \frac{16j^2 m^2}{r^2} + \frac{8jml}{r^2} + \frac{2ml^2}{r^4}. \tag{4}$$

In the above equations, each σ corresponds to the right-invariant one-forms of $SU(2)$ with the Euler angles (θ, ϕ, ψ) . Namely, we have

$$\sigma_1 = \sin \phi d\theta - \cos \phi \sin \theta d\psi, \tag{5}$$

$$\sigma_2 = \cos \phi d\theta + \sin \phi \sin \theta d\psi, \tag{6}$$

$$\sigma_3 = d\phi + \cos \theta d\psi. \tag{7}$$

It is possible to rewrite the metric (1) in different forms [32].

(i) Expanded form:

$$ds^2 = -f(r)dt^2 - 2a(r)dt\sigma_3 + g(r)\sigma_3^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4} (\sigma_1^2 + \sigma_2^2), \tag{8}$$

where

$$g(r) = \frac{r^2V(r) - 4a^2(r)}{4f(r)} = -j^2r^4 + \frac{1 - 8mj^2}{4}r^2 + \frac{ml^2}{2r^2}. \tag{9}$$

(ii) Lapse-shift form:

$$ds^2 = -N^2dt^2 + g(r)(\sigma_3 - \frac{a(r)}{g(r)}dt)^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4} (\sigma_1^2 + \sigma_2^2), \tag{10}$$

where

$$N^2 = f(r) + \frac{a^2(r)}{g(r)} = \frac{r^2V(r)}{4g(r)}. \tag{11}$$

It is worth noting that l stands for the rotation parameter of the black hole, j is known as the Gödel parameter, and m stands for the mass parameter of the black hole. The zero limit of the j and l reduces the metric (1) to the 5-dimensional Schwarzschild black hole. Besides, in the case of $m = l = 0$ the metric reduces to that of 5-dimensional Gödel universe [75]. Metric in (1) is nothing but the Schwarzschild-Gödel black hole when $l = 0$. Furthermore, metric in (1) is regular at the horizons and the tensor scalars become singular only at $r = 0$.

Employing the Wick-rotation method, it was shown that the Hawking temperature of the SDKGBH reads [76]

$$T_H = \frac{m [(1 - 8j^2m - 4ml)r_+^2 - 2l^2]}{\pi r_+^3 \sqrt{(1 - 8j^2m)r_+^4 + 2ml^2 - 4j^2r_+^6}}, \tag{12}$$

in which r_+ denotes the event horizon of the black hole. In the following section, we shall attempt to re-derive (12) by considering quantum tunneling phenomenon of the massive vector particles.

3 Quantum Tunneling of Spin-1 Particles From 5DKGBH

In this section, we calculate the Hawking radiation of the spin-1 particles from the 5DKGBH spacetime. For this purpose, we firstly transform the lapse-shift form of the 5GKGBH in (10) to the Gullstrand-Painlevé coordinates [77, 78]. Then, we shall employ the Proca equation [51] with the Hamilton-Jacobi ansatz. Our computations will be in the framework of semi-classical analysis of the WKB approximation.

Proca equation for the massive spin-1 particles is given by [49–51]

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \Psi^{\nu\mu}) + \frac{m^2}{\hbar^2} \Psi^\nu = 0, \tag{13}$$

where the anti-symmetric tensor is governed by

$$\Psi_{\mu\nu} = \partial_\mu \Psi_\nu - \partial_\nu \Psi_\mu, \tag{14}$$

by which Ψ^ν is the vector field. Without loss of generality, one can simplify the equations to be obtained from (13) by redefining the metric in (10) with the dragging coordinate [32] $\chi = \phi - \Omega_H t$ in which Ω_H represents the angular velocity of locally non-rotating observers:

$$\Omega_H = \frac{d\phi}{dt} = \frac{a(r)}{g(r)}. \tag{15}$$

We are interested in tunneling of massive vector particles that have no angular momentum ($\ell = 0$) so we set $d\chi = 0$ (and for convenience also $d\psi = d\theta = 0$). Thus, we find out the following simplified metric:

$$ds^2 = -\frac{r^2 V(r)}{4g(r)} dt^2 + \frac{dr^2}{V(r)}. \tag{16}$$

To rewrite (16) in Gullstrand-Painlevé coordinates, one can use the following transformation:

$$t \rightarrow t - \frac{2\sqrt{g(r)}}{rV(r)} \sqrt{1 - V(r)} dr. \tag{17}$$

Thus, we have

$$ds^2 = -\frac{r^2 V(r)}{4g(r)} dt^2 + \frac{r}{\sqrt{g(r)}} \sqrt{1 - V(r)} dr dt + dr^2, \tag{18}$$

which can be rewritten as follows:

$$ds^2 = -F(r) dt^2 + H(r) dr dt + dr^2 \tag{19}$$

where

$$F(r) = \frac{r^2 V(r)}{4g(r)}, \tag{20}$$

$$H(r) = \frac{r}{\sqrt{g(r)}} \sqrt{1 - V(r)}. \tag{21}$$

Let us now assume an ansatz for the vector spinor that suits for the WKB approximation as follows

$$\Psi_\nu = C_\nu \exp\left(\frac{i}{\hbar} (S_0(t, r) + \hbar S_1(t, r) + \dots)\right), \tag{22}$$

where $C_\nu = \{C_1, C_2\}$ represents some arbitrary constants and the leading order action S_0 is defined as

$$S_0(t, r) = -Et + W(r) + k, \tag{23}$$

in which E and k represent the energy and complex constant, respectively.

Hence, the resulting equations obtained in the leading order \hbar are as follows:

$$\frac{2H(r)m^2 + 4C_2 F(r)m^2 - 4C_1 E W'(r) - 4C_2 E^2}{H(r)^2 + 4F(r)} = 0 \tag{24}$$

$$\frac{-4C_1 W'(r)^2 - 4C_2 E W'(r) - 4m^2 \left(-\frac{C_2 H(r)}{2} + C_1\right)}{H(r)^2 + 4F(r)} = 0 \tag{25}$$

Thus, the associated matrix of (24) and (25) becomes

$$\aleph = \begin{bmatrix} \frac{2H(r)m^2 - 4E \frac{d}{dr} W'(r)}{(H(r))^2 + 4F(r)} & \frac{4F(r)m^2 - 4E^2}{(H(r))^2 + 4F(r)} \\ \frac{-4W'(r)^2 - 4m^2}{H(r)^2 + 4F(r)} & \frac{2H(r)m^2 - 4E \frac{d}{dr} W'(r)}{(H(r))^2 + 4F(r)} \end{bmatrix} \tag{26}$$

It is fact that when the determinant of matrix of homogeneous system of linear equations is zero ($\det \aleph = 0$) [49], we have a non-trivial solution.

Based on this fact, we compute the determinant of the matrix (26) as

$$\frac{-4m^2(-H(r)^2m^2 + 4EH(r)W'(r) - 4F(r)W'(r)^2 - 4F(r)m^2 + 4E^2)}{(H(r)^2 + 4F(r))^2} = 0. \tag{27}$$

From (27), one can obtain the following integral solution for the radial function:

$$W_{\pm} = \pm \int \frac{1}{2F(r)} \left(EH(r) + \sqrt{4E^2F(r) - F(r)H(r)^2m^2 + E^2H(r)^2 - 4F(r)^2m^2} \right) dr. \tag{28}$$

Around the event horizon $[F(r_+) = 0]$, (28) reduces to

$$W_{\pm} \approx \pm \int \frac{EH(r)}{F(r)} dr. \tag{29}$$

It is clear from (29) that the integrand possesses a simple pole at the event horizon. To overcome this difficulty, we use the Feynman’s prescription [79]. After a straightforward calculation, one can obtain

$$W_{\pm} \approx \pm i\pi E \frac{H(r_+)}{F'(r_+)}. \tag{30}$$

Now, we have two solutions: W_+ corresponds to the radial solutions of the outgoing spin-1 particles and W_- represents the ingoing particles that move toward the 5DKGBH. Therefore, the emission and absorption probabilities of the spin-1 particles crossing the 5DKGBH’s event horizon each way become

$$P_{ems} = \exp\left(-\frac{2}{\hbar} \text{Im}S_0\right) = \exp\left[-\frac{2}{\hbar} (\text{Im}W_+ + \text{Im}k)\right], \tag{31}$$

$$P_{abs} = \exp\left(-\frac{2}{\hbar} \text{Im}S_0\right) = \exp\left[-\frac{2}{\hbar} (\text{Im}W_- + \text{Im}k)\right]. \tag{32}$$

According to our understanding of classical black holes, we should set the probability of the ingoing particle to 100%. This is possible with $\text{Im}k = -\text{Im}W_-$. On the other hand, we know from (30) that $W_- = -W_+$. Combining those information, one can read the quantum tunneling rate of the spin-1 particles as

$$\Gamma = \frac{P_{ems}}{P_{abs}} = \exp\left[-\frac{4}{\hbar} \text{Im}W_+\right]. \tag{33}$$

Recalling the Boltzmann factor $\Gamma = \exp\left(-\frac{E}{T_S}\right)$ [66], the surface temperature of the 5DKGBH can be found as

$$\begin{aligned} T_S &= \frac{F'(r_+)}{4\pi H(r_+)}, \\ &= \frac{m \left[(1 - 8j^2m - 4ml)r_+^2 - 2l^2 \right]}{\pi r_+^3 \sqrt{(1 - 8j^2m)r_+^4 + 2ml^2 - 4j^2r_+^6}}. \end{aligned} \tag{34}$$

One can immediately observe that T_S is identical to the (12), which is the Hawking temperature of the 5DKGBH computed via the method of Wick rotation.

4 Conclusion

To summarize, in this paper, we have studied the Hawking radiation of the 5DKGBH. For this purpose, we have considered the massive spin-1 particles that are quantum mechanically

allowed to emit from the 5DKGBH. We have started our analysis by considering the Proca equation (13). However, due to the lengthy Proca equations of the lapse-shift metric of the 5DKGBH (10), we have transformed the metric (10) to the dragging framework expressed in the Gullstrand-Painlevé coordinates (18). Then, with the help of the Hamilton-Jacobi ansatz of the vector field (22), we have shown how the quantum tunneling of massive spin-1 particles from the 5DKGBH results in the Hawking temperature. We also plan to extend the current work to the quantum tunneling of the massive spin-2 particles [42] from the 5DKGBH. This will be our next study in the near future.

Acknowledgments The authors are grateful to the Editor and anonymous Referees for their valuable comments and suggestions to improve the paper. This work was supported by the Chilean FONDECYT Grant No. 3170035 (AÖ).

References

1. Hawking, S.W.: *Commun. Math. Phys.* **43**, 199 (1975)
2. Gibbons, G.W., Hawking, S.W.: *Phys. Rev. D* **15**, 2752 (1977)
3. Gibbons, G.W., Hawking, S.W.: *Phys. Rev. D* **15**, 2738 (1977)
4. Kraus, P., Wilczek, F.: *Mod. Phys. Lett. A* **9**, 3713 (1994)
5. Kraus, P., Wilczek, F.: *Nucl. Phys. B* **437**, 231 (1995)
6. Parikh, M.K., Wilczek, F.: *Phys. Rev. Lett.* **85**, 5042 (2000)
7. Parikh, M.K.: *Phys. Lett. B* **546**, 189 (2002)
8. Parikh, M.K.: *Int. J. Mod. Phys. D* **13**, 2351 (2004)
9. Angheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: *J. High Energy Phys.* **05**, 014 (2005)
10. Srinivasan, K., Padmanabhan, T.: *Phys. Rev. D* **60**, 024007 (1999)
11. Shankaranarayanan, S., Srinivasan, K., Padmanabhan, T.: *Mod. Phys. Lett.* **16**, 571 (2001)
12. Akhmedov, E.T., Akhmedova, V., Singleton, D.: *Phys. Lett. B* **642**, 124 (2006)
13. Akhmedov, E.T., Akhmedova, V., Pilling, T., Singleton, D.: *Int. J. Mod. Phys. A* **22**, 1705 (2007)
14. Akhmedova, V., Pilling, T., de Gill, A., Singleton, D.: *Phys. Lett. B* **666**, 269 (2008)
15. Akhmedov, E.T., Pilling, T., Singleton, D.: *Int. J. Mod. Phys. D* **17**, 2453 (2008)
16. Akhmedova, V., Pilling, T., de Gill, A., Singleton, D.: *Phys. Lett. B* **673**, 227 (2009)
17. Kerner, R., Mann, R.B.: *Phys. Rev. D* **73**, 104010 (2006)
18. Kerner, R., Mann, R.B.: *Class. Quantum Grav.* **25**, 095014 (2008)
19. Chowdhury, B.D.: *Pramana* **70**, 593 (2008). *Pramana* **70**, 3-26 (2008)
20. Yale, A.: *Eur. Phys. J. C* **71**, 1622 (2011)
21. Singleton, D., Vagenas, E.C., Zhu, T., Ren, J.R.: *J. High Energy Phys.* **1008**, 089 (2010). Erratum-ibid. **1101**, 021 (2011)
22. Singleton, D., Vagenas, E.C., Zhu, T.: *J. High Energy Phys.* **1405**, 074 (2014)
23. Rahman, M.A., Hossain, M.I.: *Phys. Lett. B* **712**, 1 (2012)
24. Ali, M.H., Sultana, K.: *Int. J. Theor. Phys.* **53**, 4035 (2014)
25. Ali, M.H., Sultana, K.: *Int. J. Theor. Phys.* **53**, 1441 (2014)
26. Ali, M.H., Sultana, K.: *Int. J. Theor. Phys.* **52**, 4537 (2013)
27. Ali, M.H.: *Int. J. Theor. Phys.* **47**, 2203 (2008)
28. Ali, M.H.: *Class. Quantum Grav.* **24**, 5849 (2007)
29. Vanzo, L., Acquaviva, G., Di Criscienzo, R.: *Class. Quantum Grav.* **28**, 18 (2011)
30. Kerner, R., Mann, R.B.: *Phys. Lett. B* **665**, 277 (2008)
31. Yale, A., Mann, R.B.: *Phys. Lett. B* **673**, 168 (2009)
32. Kerner, R., Mann, R.B.: *Phys. Rev. D* **75**, 084022 (2007)
33. Chen, D., Wu, H., Yang, H., Yang, S.: *Int. J. Mod. Phys. A* **29**, 1430054 (2014)
34. Chen, D., Wu, H., Yang, H.: *J. Cosmol. Astropart. Phys.* **1403**, 036 (2014)
35. Tawfik, A.N., Diab, A.M.: *Rept. Prog. Phys.* **78**, 126001 (2015)
36. Faizal, M., Majumdar, B.: *Ann. Phys.* **357**, 49 (2015)
37. Sakalli, I., Ovgun, A., Mirekhtiari, S.F.: *Int. J. Geom. Meth. Mod. Phys.* **11**, 1450074 (2014)
38. Sakalli, I., Ovgun, A.: *Europhys. Lett.* **110**, 10008 (2015)
39. Sakalli, I., Ovgun, A.: *Astrophys. Space Sci.* **359**, 32 (2015)

40. Ovgun, A.: *Int. J. Theor. Phys.* **55**(6), 2919–2927 (2016)
41. Sakalli, I., Ovgun, A.: *J. Astrophys. Astron.* **37**, 21 (2016)
42. Sakalli, I., Ovgun, A.: *Eur. Phys. J. Plus* **131**, 184 (2016)
43. Jusufi, K.: *Gen. Relativ. Grav.* **47**, 124 (2015)
44. Li, X.Q.: *Phys. Lett. B* **763**, 80 (2016)
45. Sakalli, I., Halilsoy, M., Pasaoglu, H.: *Astrophys. Space Sci.* **340**, 155 (2012)
46. Mazharimousavi, S.H., Sakalli, I., Halilsoy, M.: *Phys. Lett. B* **672**, 177 (2009)
47. Sakalli, I., Halilsoy, M., Pasaoglu, H.: *Int. J. Theor. Phys.* **50**, 3212 (2011)
48. Pasaoglu, H., Sakalli, I.: *Int. J. Theor. Phys.* **48**, 3517–3525 (2009)
49. Kruglov, S.I.: *Mod. Phys. Lett. A* **29**, 1450203 (2014)
50. Kruglov, S.I.: *Phys. Int. J. Mod. A* **29**, 1450118 (2014)
51. Greiner, W., Reinhardt, J.: *Field Quantization*, pp. 141–170. Springer, Berlin (1996)
52. Particle Data Group (C. Patrignani et al.): *Chin. Phys. C* **40**, 100001 (2016)
53. Barter, W. (CERN): *Mod. Phys. Lett. A* **31**, 1630044 (2016)
54. Sakalli, I., Ovgun, A.: *J. Exp. Theor. Phys.* **121**, 404 (2015)
55. Sakalli, I., Ovgun, A.: *Eur. Phys. J. Plus* **130**, 110 (2015)
56. Sakalli, I., Ovgun, A.: *Gen. Relativ. Gravit.* **48**, 1 (2016)
57. Gursel, H., Sakalli, I.: *Can. J. Phys.* **94**, 147 (2016)
58. Ovgun, A., Jusufi, K.: *Eur. Phys. J. Plus* **131**, 177 (2016)
59. Jusufi, K., Ovgun, A.: *Astrophys. Space Sci.* **361**, 207 (2016)
60. Sakalli, I., Ovgun, A.: *Astrophys. Space Sci.* **361**, 330 (2016)
61. Jusufi, K., Ovgun, A.: *Int. J. Theor. Phys.* **56**(6), 1725 (2017)
62. Sakalli, I., Ovgun, A.: *EPL*. **118**, 60006 (2017). <https://doi.org/10.1209/0295-5075/118/60006>
63. Jusufi, K.: *Europhys. Lett.* **116**, 60013 (2016)
64. Jusufi, K., Ovgun, A.: *Adv. High Energy Phys.* **2017**, 8798657 (2017)
65. Ovgun, A., Jusufi, K.: *Eur. Phys. J. Plus* **132**(7), 298 (2017)
66. Wald, R.M.: *General Relativity*. The University of Chicago Press, Chicago and London (1984)
67. Gödel, K.: *Rev. Mod. Phys.* **21**, 447 (1949)
68. Gauntlett, J.P., Gutowski, J.B., Hull, C.M., Pakis, S., Reall, H.S.: *Class. Quantum Grav.* **20**, 4587 (2003)
69. Herdeiro, C.A.R.: *Nucl. Phys. B* **665**, 189 (2003)
70. Boyda, E.K., Ganguli, S., Horava, P., Varadarajan, U.: *Phys. Rev. D* **67**, 106003 (2003)
71. Harmark, T., Takayanagi, T.: *Nucl. Phys. B* **662**, 3 (2003)
72. Blau, M., Figueroa-O'Farrill, J., Hull, C., Papadopoulos, G.: *J. High Energy Phys.* **01**, 047 (2002)
73. Berenstein, D., Maldacena, J.M., Nastase, H.: *J. High Energy Phys.* **04**, 013 (2002)
74. Gimon, E.G., Hashimoto, A.: *Phys. Rev. Lett.* **91**, 021601 (2003)
75. Gauntlett, P., Gutowski, J.B., Hull, C.M., Pakis, S., Reall, H.S.: *Class. Quantum Grav.* **20**, 4587 (2003)
76. Barnich, G., Compere, G.: *Phys. Rev. Lett.* **95**, 031302 (2005)
77. Martel, K., Poisson, E.: *Am. J. Phys.* **69**, 476 (2001)
78. Natario, J.: *Gen. Relativ. Grav.* **41**, 2579 (2009)
79. Feynman, R.P.: *The Principle of Least Action in Quantum Mechanics* (PhD thesis). Princeton University, Princeton (1942)