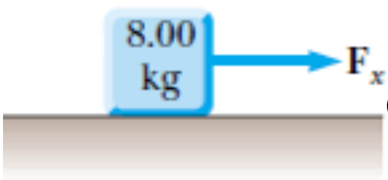
PHYS101 Final Exam - Solution Set

Department of Physics

Fall 2014/15 - January 26, 2015

Questions:

1. A block of mass 8 kg initially at rest is pulled to the right along a rough horizontal surface. Let the coefficient of static friction be $\mu_s = 0.4$ and the coefficient of kinetic friction be $\mu_k = 0.2$.



   (a) If the force pulling the block is $\vec{F} = 25\hat{i}N$ calculate the net force acting on the object, then compute the acceleration of the object and the position after $3s$.

   (b) If the force pulling the block is $\vec{F} = 50\hat{i}N$, calculate the work done by the pulling force, and the work done by the kinetic frictional force after having moved for 4m.

Solution:

(a)

$$f_{s,max} = \mu_s mg = 0.4 \times 8kg \times 9.8\frac{m}{s^2} = 31.36N > F \Longrightarrow a = 0$$

As the applied force is less than the static frictional force, the box remains at rest. Therefore $a = 0$.

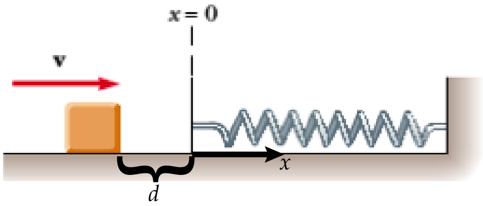
(b)

$$W = \vec{F} \cdot \vec{d} = 50\hat{i}N \cdot 4\hat{i}m = 200J$$
$$W_{fr} = \vec{F}_{fr} \cdot \vec{d} = (-\mu_k mg)\hat{i} \cdot \vec{d} = -(0.2 \times 8kg \times 9.8\frac{m}{s^2})\hat{i} \cdot (4m)\hat{i} = -62.72J$$

2. A $3kg$ block is moving on a horizontal surface with an initial velocity of $\vec{v}_0 = 2\hat{\imath}\, m/s$. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.2$. The mass hits the spring, located at distance of $d = 1.9m$ and stops. If the maximum compression of the spring is $0.1m$, calculate

   (a) the spring constant k,

   (b) the change in the kinetic energy of the object.



Solution:

(a)

$$\begin{aligned}
\Delta E_{mech} &= -f_k d \\
(K_f + U_f) - (K_i + U_i) &= -f_k d \\
\left(0 + \frac{1}{2}kx^2\right) - \left(\frac{1}{2}mv_0^2 + 0\right) &= -f_k d \\
\frac{1}{2}kx^2 &= \frac{1}{2}mv_0^2 - f_k d \\
k &= \frac{2}{x^2}\left(\frac{1}{2}mv_0^2 - \mu_k m g d\right) \\
k &= \frac{2}{(0.1m)^2}\left(\frac{1}{2}3kg\left(2\frac{m}{s}\right)^2 - 0.2 \times 3kg \times 9.8\frac{m}{s^2} \times 1m\right) = 24\frac{N}{m}
\end{aligned}$$

(b)

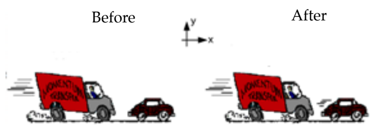
$$\Delta K = K_f - K_i = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}3kg\left(2\frac{m}{s}\right)^2 = -6J$$

3. A truck of mass $m_1 = 6000kg$ is travelling at a velocity of $\vec{v}_{1_0} = 25\hat{\imath}\, m/s$ collides with a car of mass $m_2 = 1200kg$ travelling at a velocity $\vec{v}_{2_0} = 20\hat{\imath}\, m/s$. The velocity of the truck immediately after the collision is $\vec{v}_1 = 22\hat{\imath}\, m/s$.

   (a) Calculate the velocity of the car, immediately after the collision.

   (b) If the collision lasted for $0.4s$, what is the average force exerted by the car on the truck?

   (c) Calculate the lost kinetic Energy.



Solution:

(a) Using conservation of momentum

$$\begin{aligned}
m_1 v_{1_0} + m_2 v_{2_0} &= m_1 v_1 + m_2 v_2 \\
v_2 &= \frac{m_1 v_{1_0} + m_2 v_{2_0} - m_1 v_1}{m_2} \\
&= \frac{6000kg \times 25\frac{m}{s} + 1200kg \times 20\frac{m}{s} - 6000kg \times 22\frac{m}{s}}{1200kg} = 35\frac{m}{s}
\end{aligned}$$

(b) In order to calculate the force exerted by the car on the truck, we have to consider the change of the momentum of the truck first:

$$\Delta p_{truck} = m_1 (v_1 - v_{1_0}) = 6000kg \left(22\frac{m}{s} - 25\frac{m}{s}\right) = -18000kg\,\frac{m}{s}$$

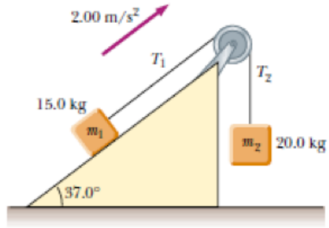
with

$$\Delta p = I = F_{avg}\Delta t \Longrightarrow F_{avg} = \frac{\Delta p}{\delta t} = \frac{-18000kg\,\frac{m}{s}}{0.4s} = -45kN$$

(c)

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right) - \left(\frac{1}{2}m_1 v_{1_0}^2 + \frac{1}{2}m_2 v_{2_0}^2\right) = -72kJ$$

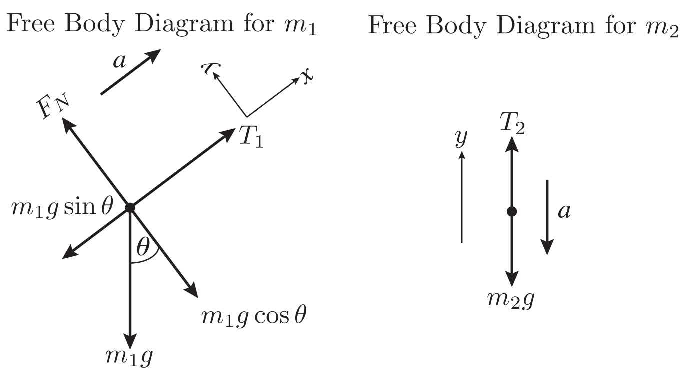
4. Two blocks having different masses $m_1 = 15kg$ and $m_2 = 20kg$ are connected by a massless string over a pulley with radius $R = 0.25m$ and moment of inertia I. Block 1 (with mass m_1) accelerates upwards the incline, that makes an angle of $\theta = 37°$ with the horizontal, at $a = 2m/s^2$.



(a) Draw the free body diagrams for the masses m_1 and m_2.
(b) Calculate the tensions in the string T_1 and T_2.
(c) Calculate the moment of inertia I of the pulley.

Solution:

(a) The free body diagrams for the masses m_1 and m_2 are:



(b) From the free-body diagram for m_1 we get

$$T_1 - m_1 g \sin\theta = m_1 a \quad (1)$$
$$F_N - m_1 g \cos theta = 0 \quad (2)$$

From (2) we get

$$F_N = m_1 g \cos\theta. \quad (3)$$

(3) in (1) gives:

$$T_1 = m_1 (g \sin\theta + a) = 15kg \left(9.8\frac{m}{s^2}\sin 37° + 2\frac{m}{s^2}\right) = 118.46N$$

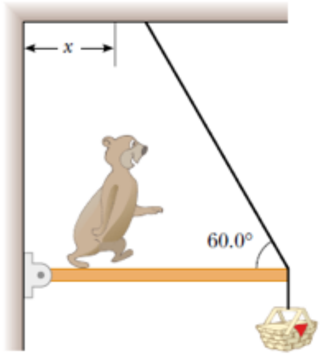
From the free-body diagram for m_2 we get

$$T_2 - m_2 g = -m_2 a \Longrightarrow T_2 = m_2(g - a) = 20kg\left(9.8\frac{m}{s^2} - 2\frac{m}{s^2}\right) = 156N \quad (4)$$

(c)

$$T_1R - T_2R = -I\alpha \Longrightarrow I = \frac{T_1R - T_2R}{-\alpha} = \frac{T_2 - T_1}{a}R^2 = \frac{156N - 118.46N}{2\frac{m}{s^2}}(0.25m)^2 = 1.19kgm^2$$

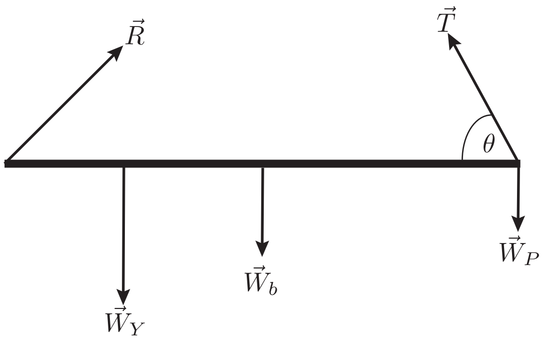
5. A uniform horizontal beam with length $\ell = 8m$ and weight $W_b = 300N$ is attached to the wall by a pin connection. Its far end is supported by a cable that makes an angle of $\theta = 60°$ with the beam. Furthermore a picknick basket of weight $W_P = 100N$ is hanging from the far end of the beam. The hungry Yogy bear with a weight of $W_Y = 800N$ is standing on a beam at a distance $d = 2m$ from the wall.



   (a) Calculate the magnitude of the tension in the cable.
   (b) Calculate the force vector exerted by the pin on the beam.

Solution:

   (a) For the solution of this problem let us first draw the free body diagram for this problem.



   So we can easily get the magnitude of the tension in the cable by considering the net torque of the system:

$$\sum \tau = -W_Y \cdot d - W_b \cdot \frac{\ell}{2} - W_P \cdot \ell + T\sin\theta \cdot \ell = 0$$

   The only unknown is the magnitude of the tension in the cable, so we solve the equation for T:

$$T = \frac{W_Y \cdot d + W_b \cdot \frac{\ell}{2} + W_P \cdot \ell}{\sin\theta \cdot \ell} = \frac{800N \cdot 2m + 300N \cdot 4m + 100N \cdot 8m}{\sin 60° \cdot 8m} = 519.6N$$

   (b) In order to calculate the force vector exerted by the pin on the beam we have to consider the components of the net force:

$$\begin{aligned} \sum F_x &= R_x - T\cos\theta = 0 \Longrightarrow R_x = T\cos\theta = 519.6N\cos 60° = 259.8N \\ \sum F_y &= R_y - W_Y - W_b - W_P + T\sin\theta = 0 \Longrightarrow \\ R_y &= W_Y + W_b + W_P - T\sin\theta = 800N + 300N + 100N - 519.6N\sin 60° = 750N \end{aligned}$$

   Therefore the force vector becomes:

$$\vec{R} = 259.8N\hat{i} + 750N\hat{j}$$