Formation of Wormholes by Dark Matter in the Galaxy Dragonfly 44

Sayeedul Islam* and Farook Rahaman†
Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India

Ali Övgün‡
Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, Chile
Physics Department, Arts and Sciences Faculty, Eastern Mediterranean University,
Famagusta, North Cyprus via Mersin 10, Turkey and
TH Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland

Mustafa Halilsoy§
Physics Department, Arts and Sciences Faculty, Eastern Mediterranean University,
Famagusta, North Cyprus via Mersin 10, Turkey
(Dated: December 29, 2017)

Recently, ultra diffuse galaxy (UDG) of Dragonfly 44 in the Coma Cluster was observed and observations of the rotational speed suggest that its mass is almost same as the mass of the Milky Way. On the other hand, interestingly, the galaxy emits only 1 % of the light emitted by the Milky Way. Then, astronomers reported that Dragonfly 44 may be made almost entirely of dark matter. In this study we try to show that the dark matter that constitutes Dragonfly 44 can form the wormhole or not. Two possible dark matter profiles are used, namely, ultra diffuse galaxy King's model and generalized Navarro-Frenk-White (NFW) dark matter profile. We have shown that King's model dark matter profile does not manage to provide wormhole whereas generalized Navarro-Frenk-White (NFW) dark matter profile is managed to find wormholes.

Keywords: Wormholes; Dark matter; Ultra Diffuse Galaxy; Dragonfly 44; NFWs Dark Matter profile

I. INTRODUCTION

Today, one of the challenging questions in a theoretical physics is the question of the existence of traversable wormholes [1–21]. Other mysteries in physics is dark matter (DM) [22–24]. Like black holes there is another miraculous objects of our universe which are wormholes. After the prediction of Einstein-Rosen bridge in 1935 [1], with a lot of theoretical evidence researchers have proposed the existence of wormholes in space-time which acts like a shortcut path to travel between any two widely separated or infinite region of the universe or between another universes in multi universe model. Structurally it looks like a tunnel (called its throat) with two mouths (most likely spheroidal). Here most interesting thing is exotic matter needed to open its throat (violates null energy conditions [2–4]. The curiosity about wormholes physics has vigorously been increased since publication of Morris and Thorne's research article where they proposed the prospect of the existence of traversable wormholes, as a solution of Einstein's field equations, that does not contain event horizon and traveler can easily move in both the regions in space-time through its straight stretch throat [2, 3]. By the existence of these hypothetical objects one can realize time machines or shortcut among faraway places of space.

Maximum mass of the universe is made up by the most mysterious substance which do not interact with the electromagnetic force, i.e. can't absorb, reflect or emit light are dark matter whose nature and composition remain overall question mark [22, 23]. Researchers hypothesize the existence of dark matter only from the gravitational effect. Observation of the spiral galaxy rotation curves is an enthralling experimental evidence for the existence of dark matter. The dark matter candidate particles are generally classified into cold, warm and hot categories. Interaction of a scalar field whose energy density is dark energy may be caused of the dark matter particle mass. Over several decades it is openly accepted that almost every galaxy contain a large amount of non luminous matter forming massive dark matter halos around the galaxy based on different lines of evidence like flat rotation curves of spiral galaxies [24] and strong lensing system [25].

The model under consideration predicts that the ultra diffuse galaxies (UDGs) space distribution for 90 globular clusters observed around Dragonfly 44 [27] has the space density:
a. King's model like [28, 29]:

$$\rho(r) \propto \kappa \left(\frac{r}{r_0}^2 + \lambda \right)^{\eta}, \tag{1}$$

where η , κ, r_0 and λ are parameters . We assume that the averaged relative speed of the galaxies in the Coma cluster is approximately equal to the velocity dispersion in the cluster $v \cong 1000$ km/s. This density profile is

---

*Electronic address: `sayeedul.jumath@gmail.com`
†Electronic address: `rahaman@associates.iucaa.in`
‡Electronic address: `ali.ovgun@pucv.cl`
§Electronic address: `mustafa.halilsoy@emu.edu.tr`

shown in fig 1.

b. Generalized Navarro-Frenk-White (NFW) profile [26]:

$$\rho(r) \propto \frac{1}{r^{\gamma}\left[1+\left(\frac{r}{r_0}\right)\right]^{3-\gamma}}, \tag{2}$$

where γ and r_0 are parameters .

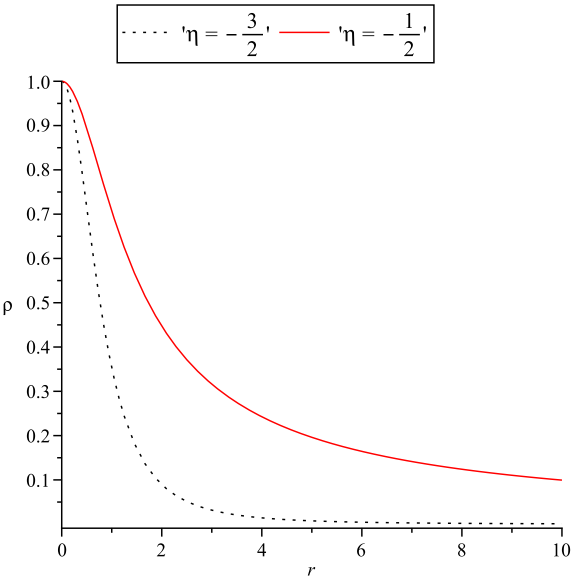

FIG. 1: Plot ρ versus r ($\kappa = 1$,$\lambda = 1$ and $r_0 = 1$).

This paper is organized as follows. Sec. II we introduces the traversable wormholes and their basic equations. In Sec. III and IV, we study the possibility of wormholes in the galaxy of Dragonfly 44 by using the UDGs dark matter profile [27–29] and NFWs dark matter profile [26],respectively. Finally, in Sec. V, we present our remarks.

II. WORMHOLES FORMULATION

The traversable wormhole space-time is given by following [2, 3]

$$ds^2 = -e^{2f(r)}dt^2+\left(1-\frac{b(r)}{r}\right)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta\, d\phi^2). \tag{3}$$

It is noted that the $b(r)$ and $f(r)$ stands for the spatial shape function, and for the redshift function, respectively. The range of the radial coordinate is from $+\infty$ to $b(r_0) = r_0$ where r$_0$, is the minimum value of the r (throat of the wormhole).

The Einstein field equations are given by

$$G^{\mu}_{\nu} = 8\pi T^{\mu}_{\nu} \tag{4}$$

where T^{μ}_{ν} and G^{μ}_{ν} are the stress-energy tensors and the Einstein tensor, respectively. Then one can calculate the non-zero Einstein tensors:

$$G^t_t = \frac{b'}{r^2}, \tag{5}$$

$$G^r_r = \frac{-b}{r^3} + 2\left(1-\frac{b}{r}\right)\frac{f'}{r}, \tag{6}$$

$$G^{\theta}_{\theta} = \left(1-\frac{b}{r}\right)\left[f''+f'^2+\frac{f'}{r}-\left(f'+\frac{1}{r}\right)\left\{\frac{b'r-b}{2r(r-b)}\right\}\right], \tag{7}$$

$$G^{\phi}_{\phi} = G^{\theta}_{\theta} \tag{8}$$

where a prime is $\frac{d}{dr}$.

DM is generally defined in the form of general anisotropic energy-momentum tensor

$$T^{\mu}_{\nu} = (\rho+p_r)u^{\mu}u_{\nu} + p_r g^{\mu}_{\nu} + (p_t - p_r)\eta^{\mu}\eta_{\nu}, \tag{9}$$

where $u^{\mu}u_{\mu} = -\frac{1}{2}\eta^{\mu}\eta_{\mu} = -1$. Note that p_t stands for the transverse pressure, p_r is the radial pressure and ρ is the energy density. A possible set of u^{μ} and η^{μ} are given by $u^{\mu} = (e^{2f(r)}, 0, 0, 0)$ and $\eta^{\mu} = (0, 0, \frac{1}{r}, \frac{1}{r\sin\theta})$. Then the stress-energy tensors T^{μ}_{ν} are calculated as follows

$$T^t_t = -\rho, \tag{10}$$

$$T^r_r = p_r, \tag{11}$$

$$T^{\theta}_{\theta} = T^{\phi}_{\phi} = p_t. \tag{12}$$

III. WORMHOLES WITH THE UDG KING'S MODEL DM

One can find the tangential velocity from the flat rotation curve for the circular stable geodesic motion in the equatorial plane as [21]

$$v^{\phi} = \sqrt{rf'}, \tag{13}$$

which is responsible to fit the flat rotational curve for the DM. Rahaman et. al observe the rotational curve profile in the DM region as follows [19, 20]

$$v^{\phi} = \alpha r \exp(-k_1 r) + \beta[1-\exp(-k_2 r)] \tag{14}$$

where α, β, k_1, and k_2 are constant positive parameters.

Using the Eqns.(13) and (14), the redshift function is obtained as follows

$$\begin{aligned} f(r) \;=\; & -\frac{\alpha^2 r}{2k_1 e^{(2k_1 r)}} - \frac{\alpha^2}{4k_1^2 e^{(2k_1 r)}} - \frac{2\alpha\beta}{k_1 e^{(k_1 r)}} \\ & + \frac{2\alpha\beta e^{(-k_1 r - k_2 r)}}{k_1+k_2} + \beta^2\ln(r) + 2\beta^2 E_i(1, k_2 r) \\ & - \beta^2 E_i(1, 2k_2 r) + D. \end{aligned} \tag{15}$$

where E_i and D are the exponential integral and integration constant, respectively. At large scales, it becomes $e^{2f(r)} = B_0 r^{(4v^{\phi})}$.

Now we discuss two cases for different values of the parameter η by using UDGs king's dark matter profile for the stability of wormholes in the galaxy Dragonfly 44.

A. The case I

For the UDGs King's DM density profile [27–29]

$$\rho(r) = \kappa \left[\left(\frac{r}{r_0} \right)^2 + \lambda \right]^{\eta}, \tag{16}$$

we assume in this case $\kappa = 1$, $\lambda = 1$ and $\eta = -3/2$. After one uses the Eq.s (13) and (14) and the UDG's dark matter density profile under the Einstein field equations, the shape function is calculated ($8\pi = 1$) as

$$b(r) = \frac{r r_0^2}{8(r^2 + r_0)} + \frac{r_o^{3/2}}{8} \tan^{-1}\left(\frac{r}{\sqrt{r_0}} \right) - \frac{r_0^3 r}{4(r^2 + r_0)^2} + C \tag{17}$$

Note that C is the integration constant and it is chosen as

$$C = r_0 - \frac{r_0^2}{8(1 + r_0)} + \frac{r_o^{3/2}}{8} \tan^{-1}(\sqrt{r_0}) + \frac{r_0^2}{4(1 + r_0)^2} \tag{18}$$

to satisfy the condition of $b(r_0) = r_0$, then it is checked the flare-out condition ($b' < 1$), where r_0 is the radius of throat.

$$b' = \rho r^2 = \frac{r_0^3 r^4}{(r^2 + r_0)^3} \tag{19}$$

which is satisfied (see fig.(2)).

Furthermore from the Eq.(10) the second derivative of b with respect to r is calculated as

$$b'' = -\frac{2 r_0^3 r (2r^2 - r_0)}{(r^2 + r_0)^4}. \tag{20}$$

The radial of pressure is showed as follows by substituting $f(r)$ and $b(r)$ into the solution (Eqns. 5- 12):

$$p_r = \frac{-b}{r^3} + 2\left(1 - \frac{b}{r}\right)\frac{f'}{r}, \tag{21}$$

$$f' = \frac{\alpha}{2r}\left(\mathrm{e}^{k_1 r}\right)^2 - 2\,\alpha\,\beta\,\mathrm{e}^{-r(k_1 + k_2)} + 2\,\frac{\alpha\,\beta}{\mathrm{e}^{k_1 r}} + \frac{\beta^2 \mathrm{e}^{-2\,k_2\,r}}{r} - 2\,\frac{\beta^2 \mathrm{e}^{-k_2\,r}}{r} + \frac{\beta^2}{r}. \tag{22}$$

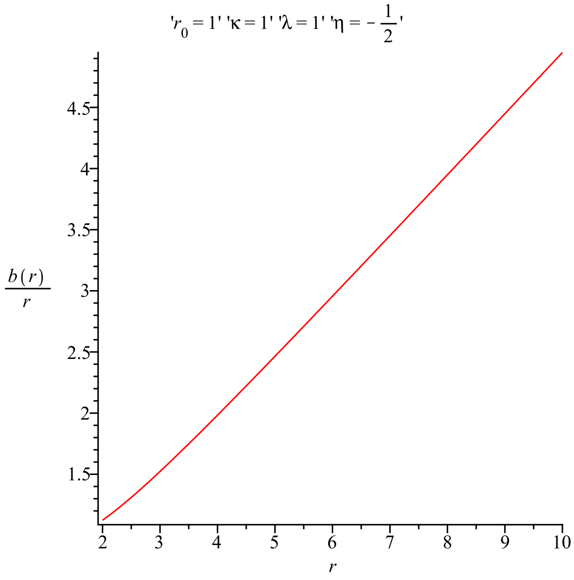

FIG. 2: The figure is shown for $b(r)/r$ versus r.

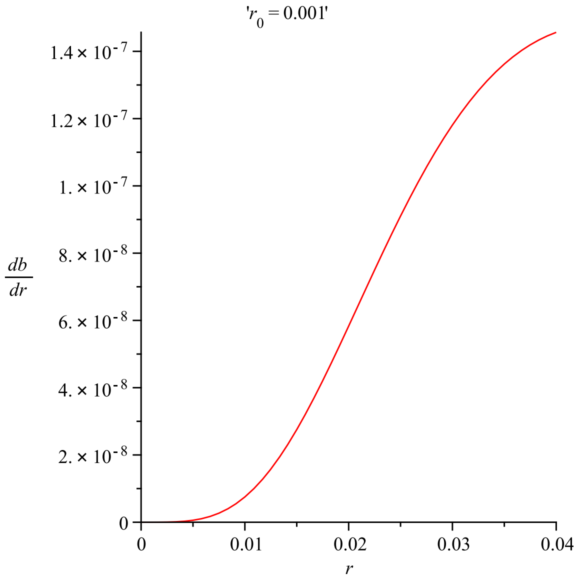

FIG. 3: The figure is shown for $b'(r)$ versus r.

It is showed in the figure (4) that the null energy condition ($\rho + p_r < 0$) is violated so that one of the essential condition for a wormhole is satisfied. However, the most important criterion for wormhole, namely, $\frac{b(r)}{r} < 1$ is violated (see fig.2). So in this case wormhole does not exists.

B. The case II

In this section we follow the same method using in case I and the shape function is calculated .

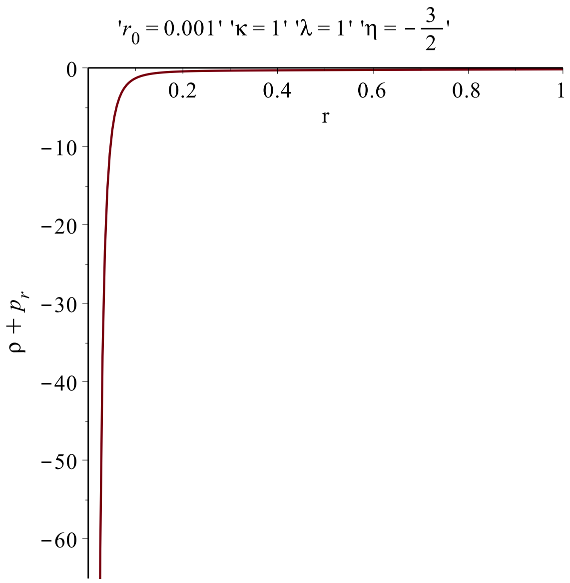

FIG. 4: Null energy condition for the case I.

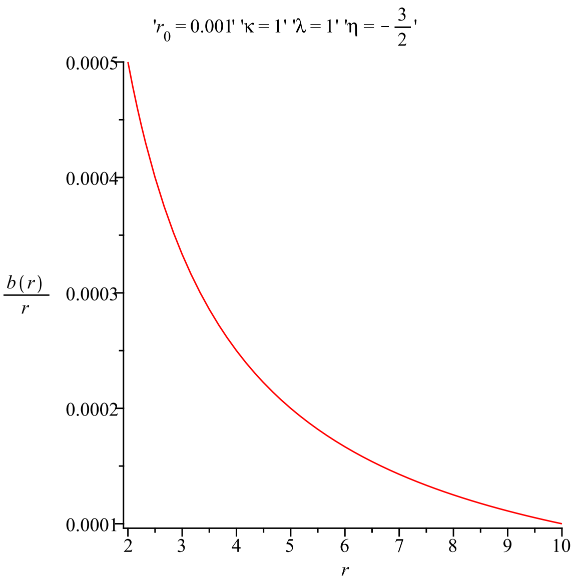

FIG. 5: The figure is shown for $b(r)/r$ versus r.

It is checked the flare-out condition ($b' < 1$) is satisfied and plotted in Fig. (6).

It is also showed in the Fig. (7) that the null energy condition ($\rho + p_r < 0$) is violated as needed to hold a wormhole. However, as in case I, $\frac{b(r)}{r} < 1$ is violated(see fig.5), so in this case wormhole does not exists. Thus King's model dark matter profile that constitutes Dragonfly 44 does not manage to provide wormhole. Now we will perform a study whether generalized NFW dark matter profile that constitutes Dragonfly 44 does manage to provide wormhole or not.

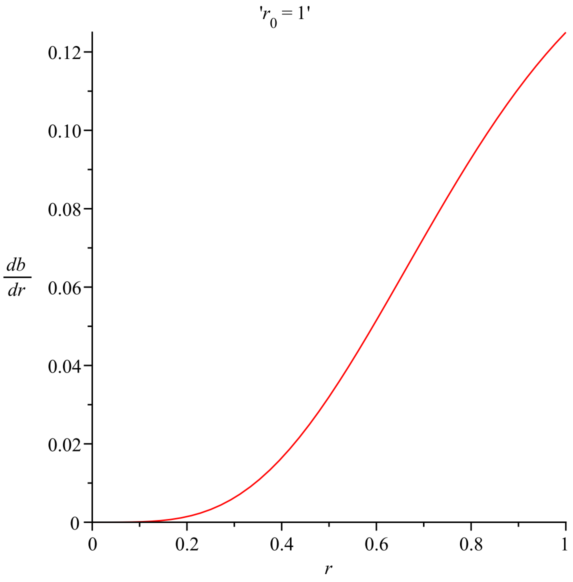

FIG. 6: The figure is shown for $b'(r)$ versus r.

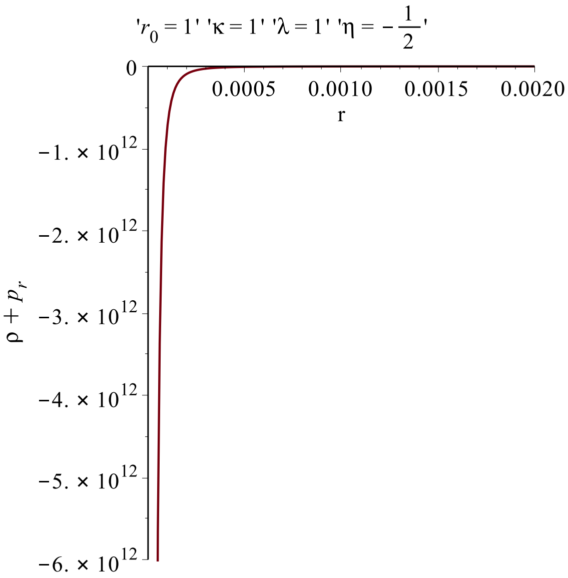

FIG. 7: Null energy condition for the case II.

IV. WORMHOLES WITH THE NFWS DM

In this section we describe the dark matter halo distribution using the following generalized Navarro-Frenk-White (NFW) profile for the existence of wormholes in the galaxy Dragonfly 44.

$$\rho(r) = \frac{1}{r^{\gamma}\left[1 + \left(\frac{r}{r_0}\right)\right]^{3-\gamma}} \qquad (23)$$

In this expression, γ is the inner slope of the profile ($\gamma = 1$ corresponds to the case of a standard NFW profile), and r_0 is the scale radius and taking variation constant as unity.

Here we have discussed three different cases for the several values of the parameter γ

A. case I

In this case we assume $r_0 = 10, \rho_0 = 0.05$ and $\gamma = 3$. After using NFW dark matter density profile under the Einstein field equations, the shape function $b(r)$ is calculated $(8\pi = 1)$ as

$$b(r) = 8\pi\rho_0\left[ln(r) + C\right] \tag{24}$$

Note that C is the integration constant and it is choosen as

$$C = \frac{r_0}{8\pi\rho_0} - lnr_0 \tag{25}$$

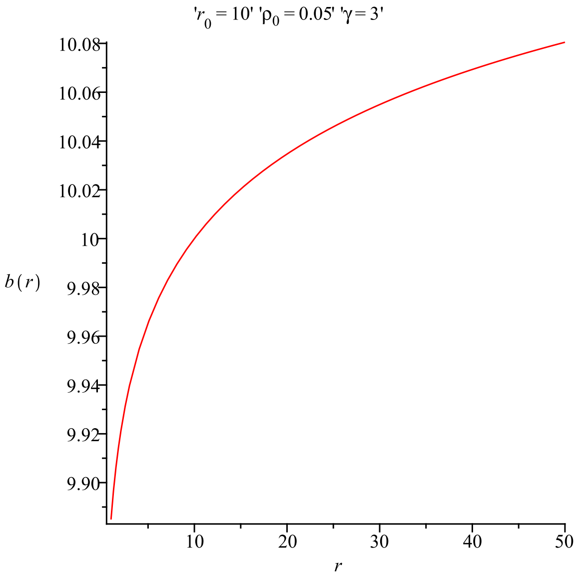

FIG. 8: The figure is shown for $b(r)$ versus r.

Here fig.(8) represents the shape function $b(r)$ and we see that the null energy condition $(\rho + p_r < 0)$ is violated (see fig.10). Also we have checked the most important flare-out condition ($b(r) - r < 0$, after the throat radius i.e. $b' < 1$) which is satisfied and is plotted in the fig.(9). Thus the generalized NFW dark matter profile that constitutes Dragonfly 44 can manage to provide wormhole.

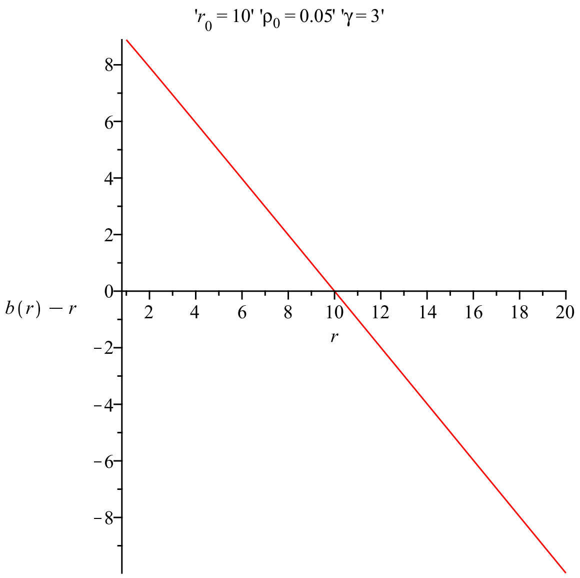

FIG. 9: The figure is shown for $b(r) - r$ versus r.

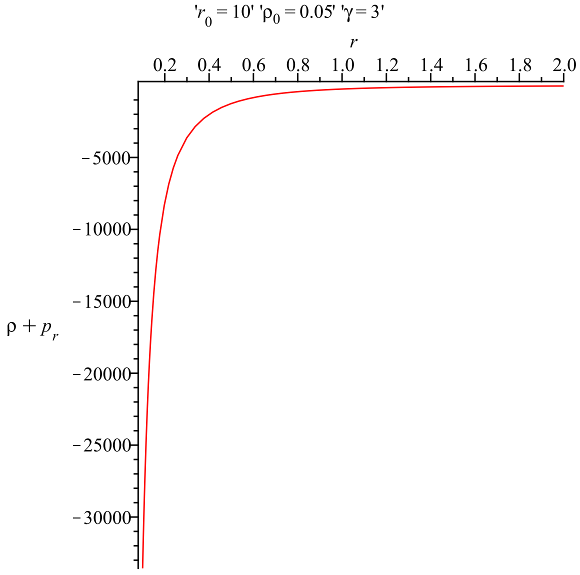

FIG. 10: Null energy condition for the case I.

B. case II

In this case we choose $r_0 = 10, \rho_0 = 0.05$ and $\gamma = 4$ After using NFW dark matter density profile under the Einstein field equations, the shape function $b(r)$ is calculated $(8\pi = 1)$ as

$$b(r) = 8\pi\rho_0\left[\frac{-1}{r} + \frac{ln(r)}{r_0} + C\right] \tag{26}$$

Note that C is the integration constant and it is choosen as

$$C = \frac{r_0}{8\pi\rho_0} + \frac{1}{r_0} - \frac{1}{r_0} ln r_0 \tag{27}$$

In this section we follow the same method used in case I and the shape function $b(r)$ is plotted in the fig. (11). To form a wormhole, the essential criteria regarding flare-out condition ($b(r) - r < 0$, after the throat radius i.e. $b' < 1$) and violation of null energy condition ($\rho + p_r < 0$) are satisfied (see fig.12 and fig.13).

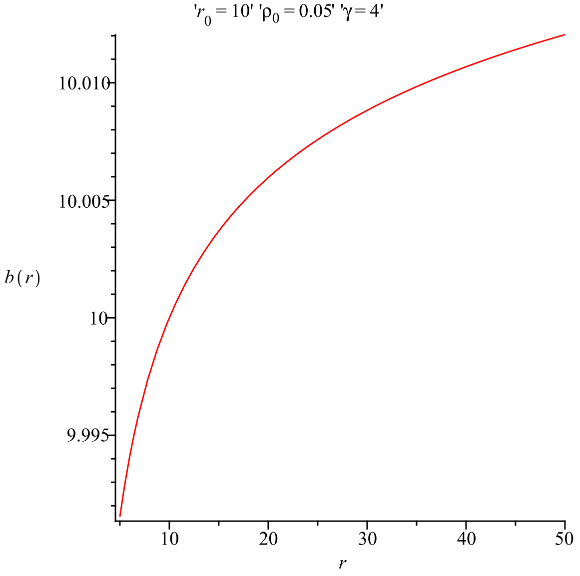

FIG. 11: The figure is shown for $b(r)$ versus r.

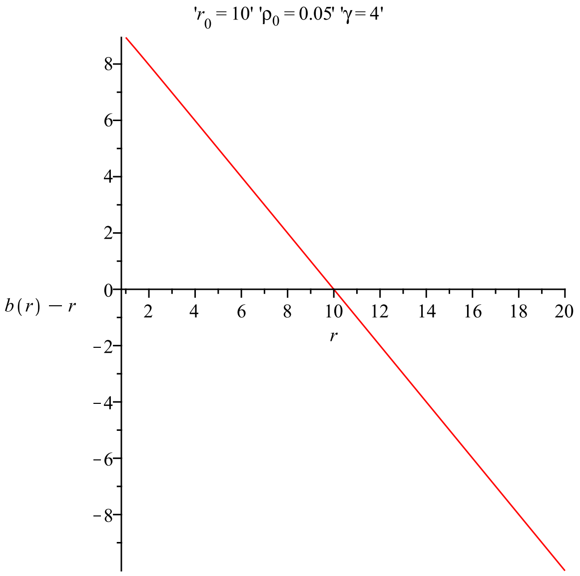

FIG. 12: The figure is shown for $b(r) - r$ versus r.

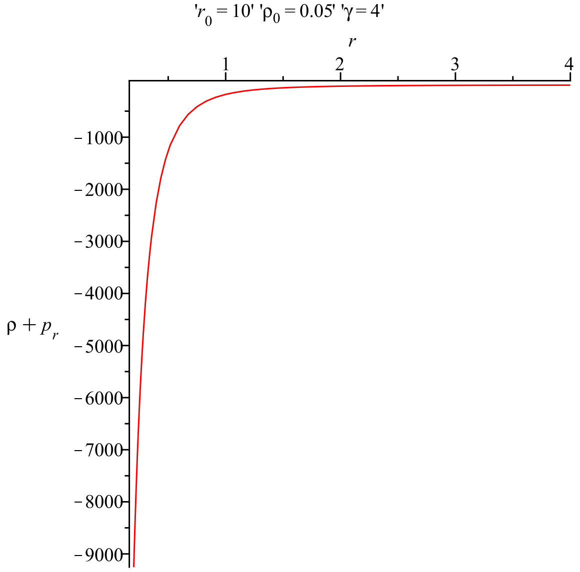

FIG. 13: Null energy condition for the case II.

C. case III

In this case we assume $r_0 = 10, \rho_0 = 0.05$ and $\gamma = 5$ After using NFW dark matter density profile under the Einstein field equations, the shape function $b(r)$ is calculated ($8\pi = 1$) as

$$b(r) = 8\pi\rho_0 \left[\frac{-2}{r r_0} + \frac{ln(r)}{r_0^2} - \frac{1}{2r^2} + C \right] \tag{28}$$

Note that C is the integration constant and it is choosen as

$$C = \frac{r_0}{8\pi\rho_0} + \frac{2}{r_0^2} - \frac{1}{r_0^2} ln r_0 + \frac{1}{2r_0^2} \tag{29}$$

Here we use the same NFW dark matter density profile like above cases and similarly the shape function $b(r)$ is calculated and plotted in the fig. (14).
In the fig. (15) and fig. (16), it is clearly shown that the flare-out condition (($b(r) - r < 0$, after the throat radius i.e. $b' < 1$)) is satisfied and null energy condition ($\rho + p_r < 0$) is violated to hold a wormhole open.

So we can claim that the generalized NFW dark matter profile that constitutes Dragonfly 44 provides wormhole.

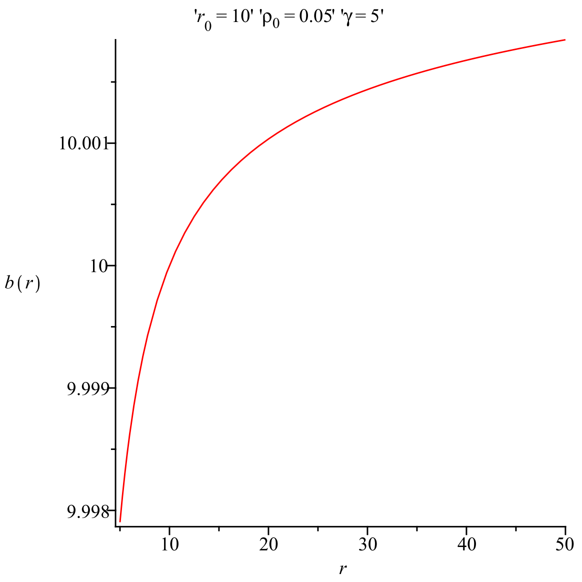

FIG. 14: The figure is shown for $b(r)$ versus r.

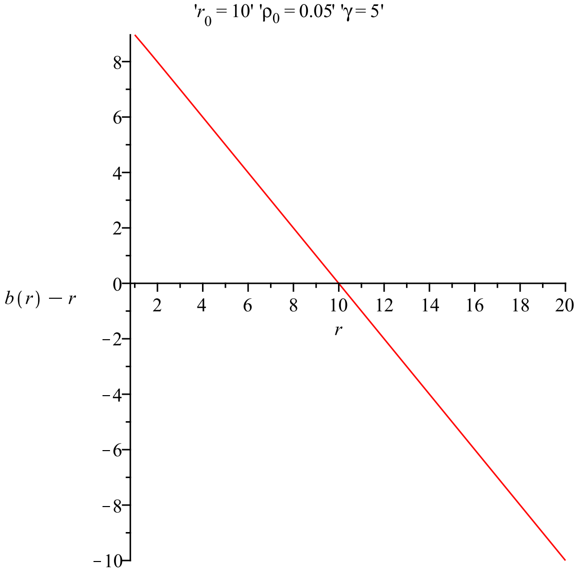

FIG. 15: The figure is shown for $b(r) - r$ versus r.

V. CONCLUSION

The presence of stable traversable wormholes is a noteworthy issue in theoretical physics. There is doubtlessly that wormholes is the most interesting objects in universe. This work is persuaded primarily by Ref.s [19–21]. In this paper, we use the UDG systems in the Coma Cluster which is known as Dragonfly 44 that astronomers reported that Dragonfly 44 may be made almost entirely of dark matter. Moreover, there is another possibility: in this study we show that this dark matter can form the wormhole, and it affects the observations. All the normal matter might be passed through this wormhole. For this purpose we firstly use the UDG profile and try to construct wormhole solution, then we repeat our calculations for the Navarro-Frenk-White (NFW) profile. We have shown that only Navarro-Frenk-White (NFW) profile provide wormhole solutions. Thus we able to find the solutions of the wormhole in the Dragonfly 44 galaxy so that the dark matter‘s halos around the Dragonfly 44 galaxy is suitable to harbor wormholes.

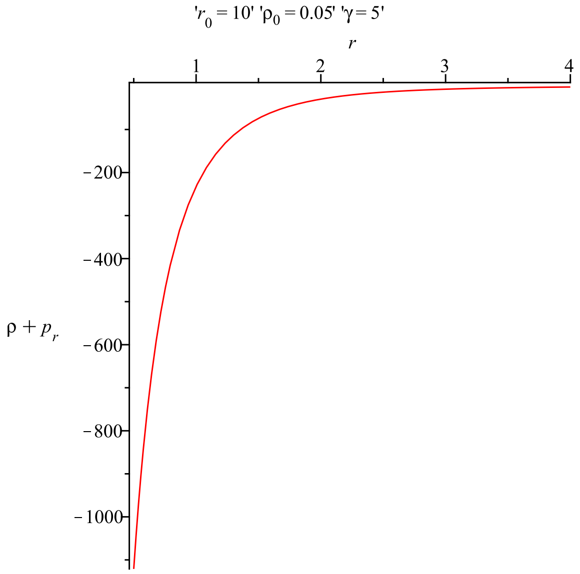

FIG. 16: Null energy condition for the case III.

Acknowledgments

This work was supported by the Chilean FONDECYT Grant No. 3170035 (AÖ). AÖ is grateful to the CERN theory (CERN-TH) division for hospitality where part of this work was done. FR would like to thank the authorities of the Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing research facilities. FR and SI are also grateful to DST-SERB and DST-INSPIRE, Govt. of India , for financial support respectively.

---

[1] A. Einstein and N. Rosen, Physical Review. **48**: 73 (1935).

[2] M.S. Moris and K.S. Thorne, Am. J. Phys. **56**, 395 (1988).

[3] M. Morris, K. S. Thorne and U. Yurtsever, Phys.Rev.Lett. **61**, 1446 (1988).
[4] M. Visser, Lorentzian Wormholes: From Einstein to Hawking, (Springer, Berlin, 1997).
[5] M. Halilsoy, A. Ovgun, S. Habib Mazharimousavi, Eur. Phys. J. C **74**, 2796 (2014).
[6] F. Rahaman, M. Kalam and S. Chakraborty, Gen. Rel. Grav. **38**, 1687 (2006).
[7] M. G. Richarte, I. G. Salako, J. P. Morais Graa, H. Moradpour and A. Övgün, Phys. Rev. D **96**, no. 8, 084022 (2017).
[8] S. Capozziello and M. Francaviglia, Gen. Rel. Grav. **40**, 357 (2008)
[9] M. S. R. Delgaty and R. B. Mann, Int. J. Mod. Phys. D **4**, 231 (1995)
[10] G. P. Perry and R. B. Mann, Gen. Rel. Grav. **24**, 305 (1992).
[11] M. Cataldo and F. Orellana, Phys. Rev. D **96**, no. 6, 064022 (2017)
[12] M. Cataldo, L. Liempi and P. Rodriguez, Phys. Lett. B **757**, 130 (2016)
[13] M. Cataldo, P. Labrana, S. del Campo, J. Crisostomo and P. Salgado, Phys. Rev. D **78**, 104006 (2008)
[14] K. Jusufi, A. Övgün and A. Banerjee, Phys. Rev. D **96**, no. 8, 084036 (2017)
[15] I. Sakalli and A. Ovgun, Eur. Phys. J. Plus **130**, no. 6, 110 (2015).
[16] I. Sakalli and A. Ovgun, Astrophys. Space Sci. **359**, no. 1, 32 (2015).
[17] A. Ovgun, Eur. Phys. J. Plus **131**, 389 (2016).
[18] S. Kar, S. Lahiri, S. SenGupta, Phys. Lett. B. **750**, 319-324 (2015).
[19] F. Rahaman, P.K.F. Kuhfittig, S. Ray, N. Islam, Eur. Phys. J. C (2014) 74:2750.
[20] F. Rahaman, P. Salucci, P.K.F. Kuhfittig, S. Ray, M. Rahaman, Ann. Phys. **350**, 561 (2014).
[21] A. Ovgun, M. Halilsoy, Astrophys Space Sci (2016) 361:214.
[22] V. Sahni, Lect. Notes Phys. **653**, 141 (2004)
[23] J. L. Feng, Ann. Rev. Astron. Astrophys. **48**, 495 (2010)
[24] T. S. V. Albada, R. Sancisi, M. Petrou and R. J. Tayler, Phil. Trans. Roy. Soc. Lond. A **320**, no. 1556, 447 (1986).
[25] C. R. Keeton, C. S. Kochanek and E. E. Falco, Astrophys. J. **509**, 561 (1998).
[26] J. F. Navarro, C. S. Frenk and S. D. M. White, Astrophys. J. **462**, 563 (1996).
[27] van Dokkum et al., Astrophys.J. **828**, no.1, L6 (2016).
[28] I. R. King, ApJL **174**, L123 (1972).
[29] A. N. Baushev, arXiv:1608.04356.