ISSN 1063-7761, Journal of Experimental and Theoretical Physics, 2015, Vol. 121, No. 3, pp. 404–407. © Pleiades Publishing, Inc., 2015.

Hawking Radiation of Spin-1 Particles from a Three-Dimensional Rotating Hairy Black Hole[1]

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Received March 27, 2015

Abstract—We study the Hawking radiation of spin-1 particles (so-called vector particles) from a three-dimensional rotating black hole with scalar hair using a Hamilton–Jacobi ansatz. Using the Proca equation in the WKB approximation, we obtain the tunneling spectrum of vector particles. We recover the standard Hawking temperature corresponding to the emission of these particles from a rotating black hole with scalar hair.

DOI: 10.1134/S1063776115090113

1. INTRODUCTION

One of the most radical predictions of general relativity is the existence of black holes. According to the seminal works of Hawking [1–3], black holes are not entirely black. That was the surprising claim made by Hawking over forty years ago. Examining the behavior of quantum fluctuations around the event horizon of a black hole, Hawking substantiated the theory that black holes emit thermal radiation, with a constant temperature (so-called Hawking temperature) directly proportional to the surface gravity κ, which is the gravitational acceleration experienced at the black hole horizon:

$$T_H = \frac{\hbar\kappa}{2\pi}, \tag{1}$$

where the system of units with $c = G = k_B = 1$ is used. The works of Hawking and Bekenstein [4] and of others [5–15], rederiving T_H in various ways, bring together the normally disparate areas: general relativity, quantum mechanics, and thermodynamics. The enthusiasm for understanding the underlying coordinations between these subjects of physics creates ample motivation for the study of Hawking radiation (see, e.g., [16–27] and references therein).

Quantum fluctuations create a virtual particle pair near the black hole horizon. While the particle with negative energy tunnels into the horizon (absorption), the other, having positive energy, flies away to the spatial infinity (emission) and produces Hawking radiation. In the WKB approximation for the emission and absorption probabilities of the tunneling particles, the tunneling rate Γ is [12, 28, 29]

$$\Gamma = \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \exp(-2\,\mathrm{Im}S)\exp\left(-\frac{E_{\text{net}}}{T}\right), \tag{2}$$

where S is the action of the classically forbidden trajectory of a tunneling particle, which has the net energy E_{net} and temperature T. One of the methods for finding S is the Hamilton–Jacobi method. This method is generally implemented by substituting a suitable ansatz, consistent with the symmetries of the space-time, in the relativistic Hamilton–Jacobi equation. The resulting radial integral always has a pole located at the event horizon. However, using the residue theory, the associated pole can be analytically avoided [30].

Recently, in the framework of the Hamilton–Jacobi method, the Hawking radiation of spin-1 particles described by the Proca equation in $3D$ nonrotating static black holes was studied by Kruglov [28]. These spin-1 particles are in fact vector particles like the Z and $W^{\pm}$ bosons, and they play a significant role in the Standard Model [31]. Based on Kruglov's study [28], Chen et al. [32] very recently investigated the Hawking radiation of these bosons in the rotating BTZ geometry. Here, similarly to [28, 32], our aim is to study the Hawking radiation of the vector particles in a three-dimensional ($3D$) rotating black hole with the scalar hairs [33–36]. These black holes are solutions of the action in $3D$ Einstein gravity that is nonminimally coupled to a scalar field ϕ. In the limit $\phi = 0$, the rotating black hole with the scalar hairs is nothing but a rotating BTZ black hole [33, 37].

This paper is organized as follows. In Section 2, we introduce the geometrical and thermodynamical features of the $3D$ rotating black hole with the scalar hairs space-time. In Section 3, we study the Proca equation for a massive boson in this geometry. We then use the

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Hamilton–Jacobi method with the separation-of-variables technique to obtain the Hawking radiation of the rotating black hole with the scalar hairs. Finally, in Section 4, we present our remarks.

2. 3D ROTATING BLACK HOLE WITH THE SCALAR HAIRS SPACE-TIME

The action in a 3D Einstein gravity with a nonminimally coupled scalar field is given by [33]

$$\mathcal{J} = \frac{1}{2}\int d^3x\sqrt{-g} \times [R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \xi R\phi^2 - 2V(\phi)], \tag{3}$$

where the coupling strength ξ between gravity and the scalar field is 1/8. Furthermore, the scalar potential $V(\phi)$ is

$$V(\phi) = -\Lambda + \frac{1}{512}\left(\Lambda + \frac{\beta}{B^2}\right)\phi^6 + \frac{1}{512}\frac{a^2}{B^4}\frac{(\phi^6 - 40\phi^4 + 640\phi^2 - 4608)\phi^{10}}{(\phi^2 - 8)^5}, \tag{4}$$

where the parameters β, B, and a are integration constants, and Λ is the cosmological constant. The line element of the rotating black hole with the scalar hairs is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2[d\theta + \omega(r)dt]^2, \tag{5}$$

with the metric functions

$$f(r) = -M\left(1 + \frac{2B}{3r}\right) + r^2\Lambda + \frac{(3r + 2B)^2J^2}{36r^4}, \tag{6}$$

$$\omega(r) = -\frac{(3r + 2B)J}{6r^3}, \tag{7}$$

where J is the angular momentum of the black hole. The scalar field is represented by

$$\phi = \pm\sqrt{\frac{8B}{r + B}}. \tag{8}$$

It is worth noting that the rotating black hole with the scalar hairs can be reduced to the rotating BTZ black hole solution when $B = 0$ [33–36]. Following [35, 36], we can see that the mass, the Hawking temperature, the Bekenstein–Hawking entropy, and the angular velocity of the particle at the horizon of this black hole are given by

$$M = \frac{J^2l^2(2B + 3r_+)^2 + 36r_+^6}{12l^2r_+^3(2B + 3r_+)}, \tag{9}$$

$$T_H = \frac{f(r_+)}{4\pi} = \frac{(B + r_+)[36r_+^6 - J^2l^2(2B + 3r_+)^2]}{24\pi l^2r_+^5(2B + 3r_+)}, \tag{10}$$

$$S_{BH} = \frac{A_H}{4G}[1 - \xi\phi^2(r_+)] = \frac{4\pi r_+^2}{B + r_+}, \tag{11}$$

$$\Omega_H = -\omega(r_+) = \frac{(3r_+ + 2B)J}{6r_+^3}, \tag{12}$$

where $\Lambda = 1/l^2$, and r_+ is referred to as the event horizon of the black hole. In order to find r_+ values, we impose the condition $f(r_+) = 0$, which yields a particular cubic equation. The solutions of that cubic equation are also given in detail in [35]. We can verify that the first law of thermodynamics

$$dM = T_HdS_{BH} + \Omega_HdJ \tag{13}$$

holds. On the other hand, calculating the specific heat using

$$C_J = T_H\left(\frac{\partial S_{BH}}{\partial T_H}\right)_J$$

proves that the rotating black hole with the scalar hairs is locally stable when $r_+ > r_{\text{ext}}$. Here, r_{ext} is the radius of an extremal rotating black hole with the scalar hairs that yields $T_H = 0$ [35].

3. HAWKING RADIATION OF SPIN-1 PARTICLES FROM AN ROTATING BLACK HOLE WITH THE SCALAR HAIRS

As described in [28], the Proca equation for massive vector particles having the wave function Φ is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\Phi^{\nu\mu}) + \frac{m^2}{\hbar^2}\Phi^\nu = 0, \tag{14}$$

where

$$\Phi_{\nu\mu} = \partial_\nu\Phi_\mu - \partial_\mu\Phi_\nu. \tag{15}$$

We choose the vector function in the form

$$\Phi_\nu = (c_0, c_1, c_2)\exp\left[\frac{i}{\hbar}S(t, r, \theta)\right], \tag{16}$$

and assume that the action is given by

$$S(t, r, \theta) = S_0(t, r, \theta) + \hbar S_1(t, r, \theta) + \hbar^2S_2(t, r, \theta) + \ldots \tag{17}$$

According to the WKB approximation, we can further set

$$S_0(t, r, \theta) = -Et + \mathcal{L}(r) + j\theta + \Bbbk, \tag{18}$$

where E and j are the energy and angular momentum of the spin-1 particles, and $\Bbbk$ is a (complex) constant. Substituting Eqs. (15)–(18) in Eq. (14) and considering the leading order in $\hbar$, we obtain an equation for a 3×3 matrix, denoted by Ξ: $\Xi(c_1, c_2, c_3)^T = 0$ (the superscript T means the transposition). The nonzero components of Ξ are

$$\begin{aligned}
\Xi_{11} &= A_1 - j^2, \\
\Xi_{12} &= \Xi_{21} = -A_2\partial_r\mathcal{L}(r), \\
\Xi_{13} &= \Xi_{31} = A_1\omega(r) - jE, \\
\Xi_{22} &= (m^2r^2 + j^2)f(r) - A_2, \\
\Xi_{23} &= \Xi_{32} = \partial_r\mathcal{L}(r)[A_2\omega(r)^2 - jf(r)^2], \\
\Xi_{33} &= \frac{A_1}{r^2}[f(r) - \omega(r)^2r^2] - E^2,
\end{aligned} \tag{19}$$

where

$$A_1 = r^2\{m^2 + f(r)[\partial_r\mathcal{L}(r)]^2\}, \tag{20}$$

$$A_2 = r^2f(r)[E + j\omega(r)]. \tag{21}$$

Using the fact that any homogeneous system of linear Eqs. (19) admits a nontrivial solution if and only if $\det\Xi = 0$, we obtain

$$\det\Xi = \frac{m^2}{r^6}\left[A_1 + j^2 - \frac{A_2^2}{r^2f(r)^3}\right]^2 = 0. \tag{22}$$

Solving for $\mathcal{L}(r)$ yields

$$\mathcal{L}_\pm(r) = \pm\int\sqrt{\frac{[E + \omega(r)j]^2 - f(r)(m^2 + j^2/r^2)}{f(r)^2}}dr. \tag{23}$$

We can immediately see that when $\omega(r) = 0$, this reduces to Kruglov's solution [28]. Here, $\mathcal{L}_+$ corresponds to outgoing spin-1 particles (moving away from the black hole) and $\mathcal{L}_-$ to the ingoing spin-1 particles (moving towards the black hole). The imaginary part of $\mathcal{L}_\pm(r)$ can be calculated by using the pole developed at the horizon. According to the complex path integration method via Feynman's prescription [30] (see [12] for a similar process), we have

$$\mathrm{Im}\,\mathcal{L}_\pm(r) = \pm\frac{\pi}{f'(r_+)}E_{\mathrm{net}}, \tag{24}$$

where

$$E_{\mathrm{net}} = E + E_0 = E + \omega(r_+)j = E - j\Omega_H. \tag{25}$$

Therefore, the probabilities of the vector particles crossing the horizon in the in- and out-directions become

$$\begin{aligned}
P_{\mathrm{emission}} &= \exp\left(-\frac{2}{\hbar}\mathrm{Im}S\right) \\
&= \exp\left[-\frac{2}{\hbar}(\mathrm{Im}\mathcal{L}_+ + \mathrm{Im}\mathbb{k})\right],
\end{aligned} \tag{26}$$

$$P_{\mathrm{absorption}} = \exp\left(\frac{2}{\hbar}\mathrm{Im}S\right) = \left[-\frac{2}{\hbar}(\mathrm{Im}\mathcal{L}_- + \mathrm{Im}\mathbb{k})\right]. \tag{27}$$

According to the classical definition of a black hole, any outside particle certainly falls onto the black hole. Therefore, we must have $P_{\mathrm{absorption}} = 1$, which yields $\mathrm{Im}\mathbb{k} = \mathrm{Im}\,\mathcal{L}_-$. On the other hand, $\mathcal{L}_+ = -\mathcal{L}_-$, and hence the total probability of radiating particles (as a consequence of quantum mechanics) is

$$\begin{aligned}
\Gamma = P_{\mathrm{emission}} &= \exp\left(-\frac{4}{\hbar}\mathrm{Im}\,\mathcal{L}_+\right) \\
&= \exp\left(-\frac{4\pi}{f'(r_+)}E_{\mathrm{net}}\right).
\end{aligned} \tag{28}$$

Comparing Eq. (28) with Eq. (2) we can recover the correct Hawking temperature (10) of a rotating black hole with the scalar hairs:

$$\begin{aligned}
T \equiv T_H &= \frac{f'(r_+)}{3\pi} \\
&= \frac{(B + r_+)[36r_+^6 - J^2l^2(2B + 3r_+)^2]}{24\pi l^2r_+^5(2B + 3r_+)}.
\end{aligned} \tag{29}$$

4. CONCLUSIONS

We have used the Proca equation to compute the tunneling rate of outgoing vector particles across the event horizon of an axially symmetric static rotating $3D$ rotating black hole with the scalar hairs. For this purpose, we have ignored the back-reaction effects and substituted the Hamilton–Jacobi ansatz in the associated Proca equations. In deriving the tunneling rate in the framework of the WKB approximation, the calculation of the imaginary part of the action was the most important point. Using the complex path integration technique, we have shown that the tunneling rate is given by Eq. (26). This result allows us to recover the standard Hawking temperature for a rotating black hole with the scalar hairs.

Finally, studying vector particles in higher-dimensional black holes may reveal more information compared to the present case.

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