Particle Acceleration by Static Black Holes in a Model of f(R) Gravity

Article *in* Canadian Journal of Physics · April 2017

DOI: 10.1139/cjp-2017-0138

Mustafa Halilsoy
Eastern Mediterranean University

Ali Övgün
Eastern Mediterranean University

NRC Research Press

ARTICLE

Particle acceleration by static black holes in a model of $f(R)$ gravity

M. Halilsoy and A. Ovgun

Abstract: Particle collisions are considered within the context of $f(R)$ gravity described by $f(R) = R + 2\alpha\sqrt{R}$, where R stands for the Ricci scalar and α is a nonzero constant. The center of mass energy of head-on colliding particles moving in opposite radial directions near the naked singularity or horizon are considered. Collision of particles in the same direction near the event horizon yields finite energy while the energy of oppositely moving particles grows unbounded. Addition of a cosmological constant does not change this feature. Collision of a massless outgoing photon with an infalling particle and collision of two oppositely moving photons following null-geodesics are also taken into account.

Key words: motion of particles, black holes, $f(R)$ gravity.

Résumé : Nous étudions les collisions de particules dans le contexte d'une gravité $f(R)$ décrite par $f(R) = R + 2\alpha\sqrt{R}$ où R est le scalaire de Ricci et α est une constante non nulle. Nous considérons le cas où l'énergie du centre de masse pour des collisions frontales de particules se déplaçant en directions radiales opposées près de la singularité (horizon) nue. Les collisions de particules dans la même direction près de l'horizon donnent une énergie finie, alors que dans le cas de particules se déplaçant en directions opposées, leur énergie croît sans limite. L'addition d'une constante cosmologique n'y change rien. Nous examinons aussi les cas d'une collision entre un photon sans masse sortant avec une particule tombant dans la singularité et celui d'une collision entre deux photons se déplaçant dans deux directions opposées sur des géodésiques nulles. [Traduit par la Rédaction]

Mots-clés : mouvement de particules, trous noirs, gravité $f(R)$, collision de particules, accélération de particules.

1. Introduction

In particle accelerators, physicists routinely accelerate elementary particles and bring them to collision. Banados, Silk, and West (BSW) [1] proposed a scenario where black holes may act as particle accelerators and first showed that the collision of geodesic particles in the vicinity of a black hole horizon yields a total unbounded center of mass (CM) energy. This amounts to a natural collision process similar to the artificially tested process in the high-energy laboratory at CERN. The difference is that the latter is under strict human control, albeit a bit too expensive process, whereas the former one is free of charge, occurring in the cosmos frequently as an ordinary event. Not only do black holes create a similar BSW effect, but so do naked singularities, as well as the throat regions of wormholes [2]. Rotating black holes and wormholes act more efficiently than static ones to yield a high CM energy [3–12]. Another aspect of the BSW effect is that it occurs irrespective of the dimensionality of space–time or the nature of the underlying theory. That is, even in lower or higher dimensions of 3+1-space–time we can have an accelerator effect [13–37]. Collision must take place near the horizon of the formed black hole or naked singularity so that the particles get a boost from the unlimited attraction or repulsion [1, 38–40].

The main aim of the paper is to study the BSW process, which is useful to detect relic cold dark matter particles that are located around the black holes. Massive dark matter particles collide and the CM energy may reach arbitrarily high energies [1]. In our earlier study [35] we showed that modification of the Einstein gravity, Horava–Lifshitz gravity, does not always lead to infinite CM energy. Since the BSW effect was found, this paper is the first time that the possibility of BSW effects has been investigated in the modified Einstein theory known as the $f(R)$ gravity with static, diagonal metrics [41]. In this theory, the Einstein–Hilbert action characterized by the Ricci scalar (R) is extended to cover an arbitrary function of R. Theoretically, such a concept has an infinite number of extensions, which are to be severely restricted by experimental tests. Naturally any higher power of R hosts higher order derivatives of the metric and expectedly obtaining exact solutions is not an easy task at all. The solution for $f(R)$ gravity that we shall consider in this study is a static one with $f(R) = R + 2\alpha\sqrt{R - 4\Lambda} - 2\Lambda$, in which the constant $\alpha \neq 0$, so that our model of $f(R)$ has no vacuum Einstein limit. By comparison with the Schwarzchild – de Sitter line element, the second integration constant Λ can be interpreted as a cosmological constant. Because it is expected that most rotating black holes would have faced the BSW effect, in fact modification of the Einstein theory changes the structure of the black hole and the BSW effect in some cases occurs and dark matter particles are accelerated. We concentrate here on our main new result, namely, computation of the limiting energy for $f(R)$ black holes. In this paper we also address the issue of CM of high-energy collisions in the absence of an event horizon and near the naked singularity.

We consider first the $\alpha < 0$ case, which represents a black hole in which collision of two infalling particles takes place near the event horizon. Oppositely moving particles near the event horizon does yield BSW effect; however, the physical situation prohibits the existence of outgoing particles from the event horizon [36, 37]. For that reason we base our argument on some physical processes that involve decay or disintegration of particles outside the

Received 24 February 2017. Accepted 24 April 2017.

M. Halilsoy. Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey.
A. Ovgun. Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey; Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaíso, Chile.
Corresponding author: Ali Ovgun (email: ali.ovgun@emu.edu.tr).

horizon. Once such a process is assumed valid, there will be outgoing as well as infalling particles in the vicinity of the black hole. As a result the existence of outgoing particles or photons will naturally invite the process of collision with different infalling particles or photons. Next, we consider the case $\alpha > 0$, as a naked singularity at $r = 0$ and the collision of two oppositely moving particles near $r \approx 0$. It is found that because of the physical constraints, such as real momenta, no unbounded CM energy arises from collisions in the vicinity of naked singularity. We note that the outgoing particle may be attributed to the repulsive effect of the naked singularity, which reverses or rebounds the particles and photons from $r \approx 0$. We cite as an example the case of negative-mass Schwarzschild metric, which gives rise to repulsive gravitational effect. Particles or photons turning outward can naturally make geodesic collisions with incoming particles.

The paper is organized as follows: Sect. 2 summarizes static spherically symmetric black holes and naked singularities in a model of $f(R)$ gravity. Collision of particles between outgoing and infalling particles near the naked singularities and event horizons are analysed in Sect. 3. Section 4 considers collisions involving photons, both outgoing and infalling. We complete the paper with our conclusion in Sect. 5.

2. Specific black hole or naked singularity

The action of general, sourceless $f(R)$ gravity theories in four dimensions is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) \tag{1}$$

in which $f(R)$ is the function of the scalar curvature R, and g stands for the determinant of the metric tensor [41].

The spherically symmetric line element is given by

$$ds^2 = -Adt^2 + \frac{1}{A}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2}$$

in which the function of $f(R)$

$$f(R) = R + 2\alpha\sqrt{R} \tag{3}$$

with the constant $\alpha \neq 0$, yields the solution

$$A(r) = \frac{1}{2} + \frac{1}{3\alpha r} \tag{4}$$

with the Ricci scalar

$$R = \frac{1}{r^2} \tag{5}$$

Inclusion of a cosmological constant Λ yields the function $f(R)$ as follows [42]:

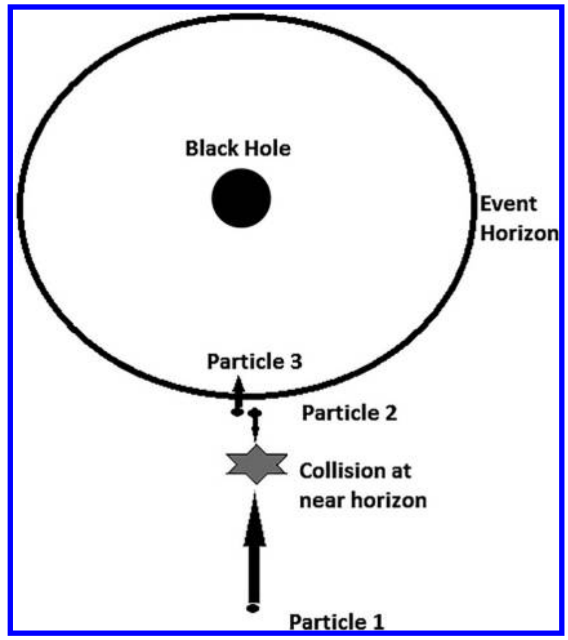
$$f(R) = R + 2\alpha\sqrt{R - 4\Lambda} - 2\Lambda \tag{6}$$

and the corresponding metric function of $A(r)$ is

$$A(r) = \frac{1}{2} + \frac{1}{3\alpha r} - \frac{\Lambda}{3}r^2 \tag{7}$$

with the scalar curvature

Fig. 1. Particle collision for which the CM energy can be very large. (Particles 1 and 3 are infalling while particle 2 is outgoing.)



$$R = \frac{1}{r^2} + 4\Lambda \tag{8}$$

Note that $\alpha \neq 0$ is already revealed by the metric function $A(r)$. In the sequel for both cases, $\Lambda = 0$ and $\Lambda \neq 0$, we shall investigate the possibility of BSW effect. Lastly, for the case of $\alpha > 0$ ($\Lambda = 0$), which corresponds to a naked singular solution at $r = 0$, we shall search for the collider effect. For the case of naked singularity, the metric function is calculated (let $\Lambda = 0$) as follows:

$$A = \frac{1}{2} + \frac{1}{3\alpha r} \tag{9}$$

Obviously in the two parametric solutions employed, Λ is a dispensable parameter whereas α is not. That is, our choice of $f(R)$ gravity lacks Einstein's general relativity limit. With deliberation we have made such a choice to see the significance of the BSW effect in a $f(R)$ model that is not connected with the general relativity. This is precisely the case with $\alpha \neq 0$.

3. Particle collision near $f(R)$ black hole and naked singularity

First, we wish to check the role of the event horizon when particles collide in case our metric is static or diagonal in $f(R)$ gravity. For different cases we investigate the CM energy for the collision, four-dimensional (4D) velocity components of the colliding particles in the background of the 4D $f(R)$ black holes by taking the radial motion on equatorial plane ($\theta = \pi/2$) (Fig. 1).

Our Lagrangian is chosen by

$$L = \frac{1}{2}\left(-A\dot{t}^2 + \frac{1}{A}\dot{r}^2 + r^2\dot{\varphi}^2\right) \tag{10}$$

in which a dot implies derivative with respect to proper time. The velocities follow as

$$u^t = \dot{t} = \frac{E}{A} \tag{11}$$

and

$$u^\varphi = \dot{\varphi} = \frac{L}{r^2} \tag{12}$$

where E and L are the energy and angular momentum constants, respectively. By using the normalization condition ($u \cdot u = -1$), it is found that the radial velocity is

$$u^r = \dot{r} = \pm\sqrt{E^2 - A\left(1 + \frac{L^2}{r^2}\right)} \tag{13}$$

and clearly we are interested in time-like geodesics. We proceed now to present the CM energy of two particles with four-velocities u_1^μ and u_2^μ. We assume that both have rest mass $m_0 = 1$. The CM energy is given by,

$$E_{\rm cm} = \sqrt{2}\sqrt{\left(1 - g_{\mu\nu}u_1^\mu u_2^\nu\right)} \tag{14}$$

so that it can be expressed as

$$\frac{E_{\rm cm}^2}{2} = 1 + \frac{E_1E_2}{A} - \kappa\frac{|L_1||L_2|}{r^2} - \kappa\frac{1}{A}\sqrt{E_1^2 - A\left(1 + \frac{L_1^2}{r^2}\right)} \times \sqrt{E_2^2 - A\left(1 + \frac{L_2^2}{r^2}\right)} \tag{15}$$

where $\kappa = \pm 1$ correspond to particles moving in the same direction ($\kappa = +1$) or opposite direction ($\kappa = -1$), respectively. Furthermore E_1 and E_2 and L_1 and L_2 are defined as the energy and angular momentum constants, respectively, corresponding to each particle. Upon taking the lowest order terms in the vicinity of the horizon, because $A \approx 0$, we can make the expansion

$$\sqrt{\left[E^2 - A\left(1 + \frac{L^2}{r^2}\right)\right]} \cong E\left[1 - \frac{A}{2E^2}\left(1 + \frac{L^2}{r^2}\right) + \ldots\right] \tag{16}$$

so that the CM energy of two particles is obtained as

$$\frac{E_{\rm cm}^2}{2} \cong 1 + (1-\kappa)\frac{E_1E_2}{A} - \kappa\frac{|L_1||L_2|}{r^2} + \frac{\kappa}{2}\left[\frac{E_2}{E_1}\left(1 + \frac{L_1^2}{r^2}\right) + \frac{E_1}{E_2}\left(1 + \frac{L_2^2}{r^2}\right)\right] \tag{17}$$

We investigate whether or not the BSW effect occurs for $A(r) \to 0$ whenever there is a horizon. In the case of the collision of ingoing–ingoing or outgoing–outgoing particles (i.e., motion in same directions) $\kappa = +1$, it reduces to

$$\frac{E_{\rm cm}^2}{2} \cong 1 - \frac{|L_1||L_2|}{r^2} + \frac{1}{2}\left[\frac{E_2}{E_1}\left(1 + \frac{L_1^2}{r^2}\right) + \frac{E_1}{E_2}\left(1 + \frac{L_2^2}{r^2}\right)\right] \tag{18}$$

For the case of the $f(R)$ black hole without cosmological constant where $\alpha < 0$, horizon is at $r_{\rm h} = 2/(3|\alpha|)$ and A goes to zero at the horizon of a $f(R)$ black hole; however, (18) has no A term so that it does not diverge and there is no BSW effect. On the other hand, when $\kappa = -1$ (i.e., when the particles move in opposite directions), we observe that the second term in (17) becomes unbounded. We stress once more that if the particles are moving both inward (i.e., $\kappa = +1$), there is no BSW effect in the $E_{\rm cm}^2$, in accordance with (18).

The occurrence of outgoing particles is crucial for a diverging $E_{\rm cm}^2$. Such an outgoing particle may be attributed to a decay or disintegration process in the vicinity of the horizon. While one of the particles falls into the hole its pair moves outward to collide with an infalling particle.

3.1. Particle collision near $f(R)$ black holes with a cosmological constant

The second case of interest is for the chosen $f(R)$ black hole model with a cosmological constant in which the metric function A is

$$A = \frac{-\Lambda r^2}{3} + \frac{1}{2} + \frac{1}{3\alpha r} \tag{19}$$

where the event horizon is located at

$$r_{\rm h} = \frac{\Xi}{2\alpha\Lambda} + \frac{\alpha}{\Xi} \tag{20}$$

for

$$\Xi = \left(4 + 2\alpha^2\Lambda^2\sqrt{\frac{-2\alpha^2 - 4\Lambda}{\Lambda}}\right)^{1/3} \tag{21}$$

At this point we must add that we are not interested in the other roots of $A(r) = 0$ that specify the inner horizon. It is observed that for real $r_{\rm h}$ we must have $(-2a^2/L) - 4 > 0$, which restricts the cosmological constant to the case of $\Lambda < 0$.

As in the case of $\kappa = +1$ the CM energy $E_{\rm cm}^2$ is finite. Collision of an infalling and outgoing particle $\kappa = -1$, however, does yield a BSW effect.

3.2. Particle collision near the naked singularity

There is a naked singularity for our $f(R)$ model at the location of $r = 0$, with $\alpha > 0$, where the metric function is given by (let $\Lambda = 0$)

$$A = \frac{1}{2} + \frac{1}{3\alpha r} \tag{22}$$

As calculated earlier, the collision of two particles generally is

$$\frac{E_{\rm cm}^2}{2} = 1 + \frac{E_1E_2}{A} - \kappa\frac{|L_1||L_2|}{r^2} - \kappa\frac{1}{A}\sqrt{E_1^2 - A\left(1 + \frac{L_1^2}{r^2}\right)} \times \sqrt{E_2^2 - A\left(1 + \frac{L_2^2}{r^2}\right)} \tag{23}$$

Let us note that each term under the square root must be positive. Such a constraint restricts the range of r to remain far from the naked singularity. Once r is finite, the overall CM energy also must be finite and therefore we observe no diverging result from the presence of the naked singularity. Choosing the pure radial motions (i.e., $L_1 = L_2 = 0$) also does not change the feature of the problem.

4. Collisional processes with photons

4.1. In naked singular space–time

The outgoing massless photon, presumably reflected from the naked singularity, can naturally scatter an infalling particle or vice versa. This phenomenon is analogous to a Compton-like scattering process, which was originally introduced for a photon and an electron. The null-geodesics for a photon satisfies

$$\frac{dt}{d\lambda} = \dot{t} = \frac{E_\gamma}{A} \tag{24}$$

and

$$\frac{d\varphi}{d\lambda} = \dot{\varphi} = \frac{L_\gamma}{r^2} \tag{25}$$

$$\dot{r} = \pm\sqrt{E_\gamma^2 - \frac{AL^2}{r^2}} \tag{26}$$

where λ and E_γ are an affine parameter and the photon energy, respectively. Defining $E_\gamma = \hbar\omega_0$, where ω_0 is the frequency (with the choice $\hbar = 1$) we can parametrize the energy of the photon by ω_0 alone. The center-of-mass energy of an outgoing photon and the infalling particle can be taken now as

$$E_{\rm cm}^2 = -(p^\mu + k^\mu)^2 \tag{27}$$

in which p^μ and k^μ refer to the particle and photon 4-momenta, respectively. It is needless to state that for a photon we have $k^2 = 0$. This amounts to (for $\theta = \pi/2$)

$$E_{\rm cm}^2 = m^2 - 2mg_{\mu\nu}p^\mu k^\nu \tag{28}$$

Because we have for the particle

$$p^\mu = m\left(\frac{E_1}{A}, \sqrt{E_1^2 - A\left(1 + \frac{L_1^2}{r^2}\right)}, 0, \frac{L_1}{r^2}\right) \tag{29}$$

and for the photon

$$k^\mu = \left(\frac{E_\gamma}{A}, \sqrt{E_\gamma^2 - \frac{AL_\gamma^2}{r^2}}, 0, \frac{L_\gamma}{r^2}\right) \tag{30}$$

one obtains

$$E_{\rm cm}^2 = m^2 + \frac{2mE_\gamma E_1}{A} - \frac{2\kappa m|L_1||L_\gamma|}{r^2} - \frac{2m\kappa}{A}\sqrt{E_\gamma^2 - \frac{AL_\gamma^2}{r^2}} \times \sqrt{E_1^2 - A\left(1S + \frac{L_1^2}{r^2}\right)} \tag{31}$$

which amounts to a collision process that must occur away from the singularity $r = 0$ (i.e., $E_{\rm cm}^2$ remains finite).

4.2. Photon–photon collision near the naked singularity

Let us consider the problem of collision between two photons in the naked singular space–time. The photons follow null geodesics in opposite directions and collide head on. In quantum electrodynamics, colliding energetic photons can transmute into particles. Because our analysis here is entirely classical we shall refer only to the CM energy of the yield without further specification. The CM energy of the product satisfies

$$E_{\rm cm}^2 = -\left(k_1^\mu + k_2^\mu\right)^2 = -2g_{\mu\nu}k_1^\mu k_2^\nu \tag{32}$$

where k_1 and k_2 correspond to the 4-momenta of respective photons. From the null-geodesic analysis, in the $\theta = \pi/2$ plane, we have

$$k_1^\mu = \left\{\frac{E_1}{A}, \sqrt{E_1^2 - \frac{AL_1^2}{r^2}}, 0, \frac{L_1}{r^2}\right\} \tag{33}$$

$$k_2^\mu = \left\{\frac{E_2}{A}, \sqrt{E_2^2 - \frac{AL_2^2}{r^2}}, 0, \frac{L_2}{r^2}\right\} \tag{34}$$

where E_1 and E_2 are the corresponding energies of different photons. Upon substitution into (32) we obtain

$$\frac{1}{2}E_{\rm cm}^2 = \frac{E_1E_2}{A} - \frac{\kappa}{A}\sqrt{E_1^2 - \frac{AL_1^2}{r^2}}\sqrt{E_2^2 - \frac{AL_2^2}{r^2}} - \frac{|L_1||L_\gamma|}{r^2} \tag{35}$$

in which we have to insert $\kappa = \pm 1$ to specify the parallel and anti-parallel propagation of the photons, respectively.

1. For $\kappa = +1$, which implies two parallel photons moving at the speed of light naturally don't scatter, so we observe no noticeable effect.
2. For $\kappa = -1$, however, the photons are moving in opposite directions and inevitably they collide. In classical background each naturally follows a null geodesic. Their corresponding CM energy from our foregoing analysis that $r_i^2 > AL_i^2/E_i^2$ for each $i = 1$, 2 remains also finite.

Let us comment that this is a collision of test photons on a given geometry without back-reaction effect. On the other hand, exact collision of electromagnetic shock plane waves in Einstein's gravity, as a highly nonlinear process [44, 45], is entirely different. As a result of mutual focusing, the latter develops null singularities after the collision process. Our conclusion is that, at the test level, collision of two oppositely moving photons in a naked singular space–time yields no observable effect.

4.3. Photon–photon collision near the horizon

From the analysis in Sect. 4.2 the CM energy of two photons is adapted as

$$\frac{1}{2}E_{\rm cm}^2 = \frac{E_1E_2}{A} - \frac{\kappa}{A}\sqrt{E_1^2 - \frac{AL_1^2}{r^2}}\sqrt{E_2^2 - \frac{AL_2^2}{r^2}} - \frac{|L_1||L_\gamma|}{r^2} \tag{36}$$

with the supplement that now we search the case for the limit $A \to 0$, instead of $A \to \infty$, because $r = r_{\rm h}$ = finite. We obtain

$$\frac{1}{2}E_{\rm cm}^2 \simeq \frac{E_1E_2}{A}(1 - \kappa) + (\text{finite terms})$$

which suggests that for $\kappa = -1$ (i.e., for oppositely moving photons) it yields an unbounded CM energy. For parallel photons, both ingoing, naturally $\kappa = +1$ and there is no observable effect.

How can an outgoing photon from the near-horizon region be justified because the Hawking photons remain too weak to cope or scatter with an infalling one? An outgoing photon can be created in an explosion or decay process by a physical particle before it falls into the horizon. Such an assumption yields an outgoing photon and naturally it has the chance to collide with an opposite photon and give rise to an unbounded energy. This is exactly what happens for two colliding oppositely moving electromagnetic plane waves to focus each other and create a null singularity [44, 45].

5. Conclusion

Collision of particles near black hole horizons in Einstein's general relativity (i.e., the BSW effect) has been considered in detail during recent years. Oppositely moving particle collisions near static black holes were also considered in refs. 9 and 47. Besides static, charged and rotating black holes were also investigated. In particular, rotational effects were shown from the original Penrose process long ago [46].

This has a significant role in the extraction of energy from black holes. In this paper, we investigated the idea of BSW to the modified theory known as $f(R)$ gravity. In particular we concentrated on $f(R) = R + 2\alpha\sqrt{R - 4\Lambda} - 2\Lambda$, which arises as an exact, source-free, spherically symmetric solution that the external energy–momentum tensor vanishes, but the curvature makes its own source. We can easily set $\Lambda = 0$, however, $\alpha \neq 0$ is an essential parameter of the model so that our model does not have the general relativity limit of $f(R) = R$. For $\alpha < 0$ we have the black hole while for $\alpha > 0$ we obtain a naked singularity at $r = 0$.

In the case of a black hole we show the existence of BSW effect provided that outgoing particles from some physical process are taken for granted. Collision of an infalling and outgoing particle $\kappa = -1$ near the horizon of the $f(R)$ black holes with or without a cosmological constant, however, does yield a BSW effect. Near a naked singularity, however, we observe no efficient collision to increase the CM energy unbounded. For oppositely moving particles, a similar result can also be obtained for a Compton-like process between a photon and a particle provided that they move in opposite directions. Collision of two oppositely moving photons near the naked singularity also yields no diverging CM energy. On the other hand, for oppositely moving photons, it yields an unbounded CM energy. Therefore it is clear that CM energy depends on the direction of the particles with the parameter of κ and the collision of the oppositely moving particles must be near the horizon of the black hole.

The CM energy distribution of relic cold dark matter particles colliding in lower or higher dimensions will be discussed in a future publication with comparing the observational data that give the possible excess of gamma rays observed in Fermi data at WIMP-scale energies [48]. Moreover, it is our belief that seeking an alternative model of gravity, which can lead to BSW effects, will be useful in the search for dark matter. For this purpose we will investigate the BSW process for black holes, strings or wormholes to look at the CM energy of the colliding neutral or charged particles. This is going to be our next problem in the near future.

Acknowledgements

This work was supported by the Chilean FONDECYT Grant No. 3170035 (AÖ).

References

1. M. Banados, J. Silk, and S.M. West. Phys. Rev. Lett. **103**, 111102 (2009). doi:10.1103/PhysRevLett.103.111102.
2. N. Tsukamoto and C. Bambi. Phys. Rev. D **91**, 084013 (2015). doi:10.1103/PhysRevD.91.084013.
3. B. Toshmatov, A. Abdujabbarov, B. Ahmedov, and Z. Stuchlik. Astrophys. Space Sci. **357**, 41 (2015). doi:10.1007/s10509-015-2289-4.
4. P. Pradhan. Astropart. Phys. **62**, 217 (2015). doi:10.1016/j.astropartphys.2014.09.005.
5. J. Sadeghi and B. Pourhassan. Eur. Phys. J. C, **72**, 1984 (2012). doi:10.1140/epjc/s10052-012-1984-3.
6. J. Sadeghi, B. Pourhassan, and H. Farahani. Commun. Theor. Phys. **62**, 358 (2014). doi:10.1088/0253-6102/62/3/12.
7. M. Patil and P.S. Joshi. Gen. Relativ. Gravit. **46**, 1801 (2014). doi:10.1007/s10714-014-1801-4.
8. M. Patil, P.S. Joshi, K. Nakao, M. Kimura, and T. Harada. Europhys. Lett. **110**, 30004 (2015). doi:10.1209/0295-5075/110/30004.
9. A.A. Grib and Y.V. Pavlov. Grav. Cosmol. **21**, 13 (2015). doi:10.1134/S0202289315010065.
10. Z. Stuchlik and J. Schee. Class. Quant. Grav. **30**, 075012 (2013). doi:10.1088/0264-9381/30/7/075012.
11. C. Zhong and S. Gao. JETP Lett. **94**, 589 (2011). doi:10.1134/S0021364011200161.
12. C. Liu, S. Chen, C. Ding, and J. Jian. Phys. Lett. B, **701**, 285 (2011). doi:10.1016/j.physletb.2011.05.070.
13. J.D. Schnittman. Phys. Rev. Lett. **113**, 261102 (2014). doi:10.1103/PhysRevLett.113.261102.
14. A. Zakria and M. Jamil. J. High En. Phys. **2015**(5), 147 (2015). doi:10.1007/JHEP05(2015)147.
15. M. Patil and P.S. Joshi. Phys. Rev. D, **82**, 104049 (2010). doi:10.1103/PhysRevD.82.104049.
16. S.W. Wei, Y.X. Liu, H. Guo, and Chun-E, Fu. Phys. Rev. D, **82**, 103005 (2010). doi:10.1103/PhysRevD.82.103005.
17. J.L. Said and K.Z. Adami. Phys. Rev. D, **83**, 104047 (2011). doi:10.1103/PhysRevD.83.104047.
18. T. Harada and M. Kimura. Phys. Rev. D, **83**, 084041 (2011). doi:10.1103/PhysRevD.83.084041.
19. M. Patil, P.S. Joshi, and D. Malafarina. Phys. Rev. D, **83**, 064007 (2011). doi:10.1103/PhysRevD.83.064007.
20. M. Banados, B. Hassanain, J. Silk, and S.M. West. Phys. Rev. D, **83**, 023004 (2011). doi:10.1103/PhysRevD.83.023004.
21. A. Galajinsky. Phys. Rev. D, **88**, 027505 (2013). doi:10.1103/PhysRevD.88.027505.
22. S.G. Ghosh, P. Sheoran, and M. Amir. Phys. Rev. D, **90**, 103006 (2014). doi:10.1103/PhysRevD.90.103006.
23. M. Bejger, T. Piran, M. Abramowicz, and F. Hakanson. Phys. Rev. Lett. **109**, 121101 (2012). doi:10.1103/PhysRevLett.109.121101.
24. T. Jacobson and T.P. Sotiriou. Phys. Rev. Lett. **104**, 021101 (2010). doi:10.1103/PhysRevLett.104.021101.
25. K. Lake. Phys. Rev. Lett. **104**, 211102 (2010). doi:10.1103/PhysRevLett.104.211102.
26. S.E. Perez Bergliaffa and Y.E. Chifarelli de Oliveira Nunes. Phys. Rev. D, **84**, 084006 (2011). doi:10.1103/PhysRevD.84.084006.
27. A.A. Grib and Y.V. Pavlov. Astropart. Phys. **34**, 581 (2011). doi:10.1016/j.astropartphys.2010.12.005.
28. A.A. Grib and Y.V. Pavlov. JETP Lett. **92**, 125 (2010). doi:10.1134/S0021364010150014.
29. S. Wei, Y. Liu, H. Li, and F. Chen. J. High En. Phys. **2010**(12), 066 (2010). doi:10.1007/JHEP12(2010)066.
30. M. Amir and S.G. Ghosh. J. High En. Phys. **2015**(07), 015 (2015). doi:10.1007/JHEP07(2015)015.
31. B. Pourhassan and U. Debnath. 2015. arXiv:1506.03443.
32. C. Liu, S. Chen, C. Ding, and J. Jing. Phys. Lett. B, **701**, 285 (2011). doi:10.1016/j.physletb.2011.05.070.
33. C. Ding, C. Liu, and Q. Quo. Int. J. Mod. Phys. D, **22**, 1350013 (2013). doi:10.1142/S0218271813500132.
34. P. Horava. Phys. Rev. D, **79**, 084008 (2009). doi:10.1103/PhysRevD.79.084008.
35. M. Halilsoy and A. Ovgun. Adv. High Energy Phys. **2017**, 4383617 (2017). doi:10.1155/2017/4383617.
36. E. Berti, R. Brito, and V. Cardoso. Phys. Rev. Lett. **114**, 251103 (2015). doi:10.1103/PhysRevLett.114.251103. PMID:26197116.
37. E. Leiderschneider and T. Piran. 2015. arXiv:1501.01984.
38. T. Harada and M. Kimura. Class. Quant. Grav. **31**, 243001 (2014). doi:10.1088/0264-9381/31/24/243001.
39. S. Hussain, I. Hussain, and M. Jamil. Eur. Phys. J. C, **74**, 3210 (2014). doi:10.1140/epjc/s10052-014-3210-y.
40. M. Sharif and N. Haider. Astrophys. Space Sci. **346**, 111 (2013). doi:10.1007/s10509-013-1424-3.
41. L. Sebastiani and S. Zerbini. Eur. Phys. J. C, **71**, 1591 (2011). doi:10.1140/epjc/s10052-011-1591-8.
42. Z. Amirabi, M. Halilsoy, and S.H. Mazharimousavi. Eur. Phys. J. C, **76**, 338 (2016). doi:10.1140/epjc/s10052-016-4164-z.
43. S. Habib Mazharimousavi, M. Halilsoy, and T. Tahamtan. Eur. Phys. J. C. **72**, 1851 (2012). doi:10.1140/epjc/s10052-011-1851-7.
44. P. Bell and P. Szekeres. Gen. Rel. Grav. **5**, 275 (1974). doi:10.1007/BF00770217.
45. M. Halilsoy. Phys. Rev. D, **37**, 2121 (1988). doi:10.1103/PhysRevD.37.2121.
46. R. Penrose. Nuovo Cimento J. **1**, 252 (1969).
47. M. Patil and P.S. Joshi. Phys. Rev. D, **86**, 044040 (2012). doi:10.1103/PhysRevD.86.044040.
48. M. Su and D.P. Finkbeiner. 2012. arXiv:1206.1616.