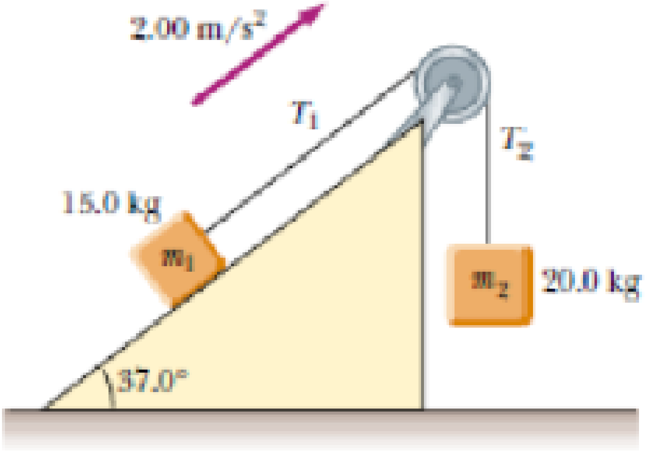
P1

Two blocks having different masses $m_1 = 15kg$ and $m_2 = 20kg$ are connected by a massless string over a pulley with radius $R = 0.25m$ and moment of inertia I. Block 1 (with mass m_1) accelerates upwards the incline, that makes an angle of $\theta = 37°$ with the horizontal, at $a = 2m/s^2$.

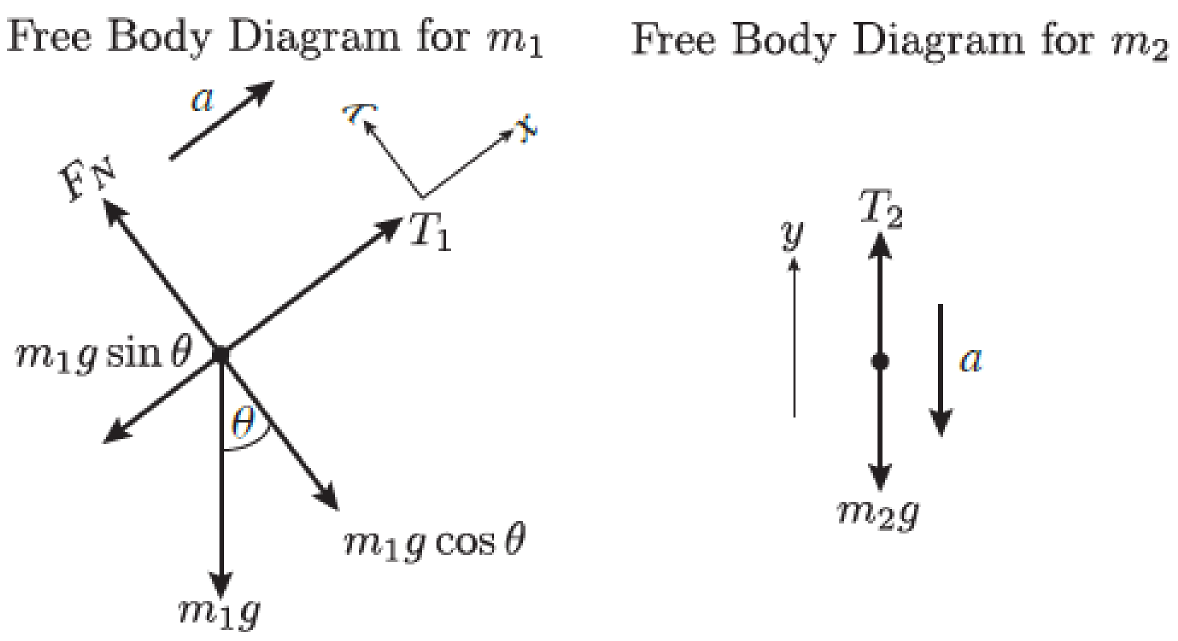


(a) Draw the free body diagrams for the masses m_1 and m_2.

(b) Calculate the tensions in the string T_1 and T_2.

(c) Calculate the moment of inertia I of the pulley.

(a) The free body diagrams for the masses m_1 and m_2 are:



(b) From the free-body diagram for m_1 we get

$$T_1 - m_1 g \sin\theta = m_1 a \tag{1}$$

$$F_N - m_1 g \cos theta = 0 \tag{2}$$

From (2) we get

$$F_N = m_1 g \cos\theta. \tag{3}$$

(3) in (1) gives:

$$T_1 = m_1 \left(g \sin\theta + a\right) = 15kg \left(9.8\frac{m}{s^2} \sin 37^\circ + 2\frac{m}{s^2}\right) = 118.46N$$

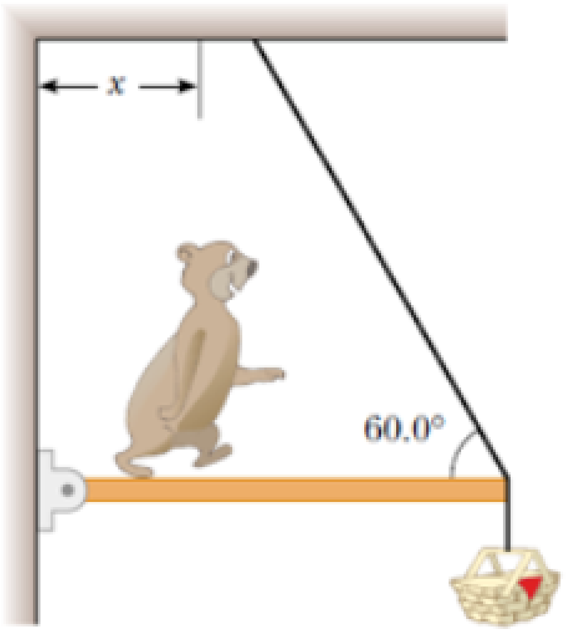
From the free-body diagram for m_2 we get

$$T_2 - m_2 g = -m_2 a \Longrightarrow T_2 = m_2(g - a) = 20kg \left(9.8\frac{m}{s^2} - 2\frac{m}{s^2}\right) = 156N \qquad (4)$$

(c)

$$T_1 R - T_2 R = -I\alpha \Longrightarrow I = \frac{T_1 R - T_2 R}{-\alpha} = \frac{T_2 - T_1}{a} R^2 = \frac{156N - 118.46N}{2\frac{m}{s^2}} (0.25m)^2 = 1.19kgm^2$$

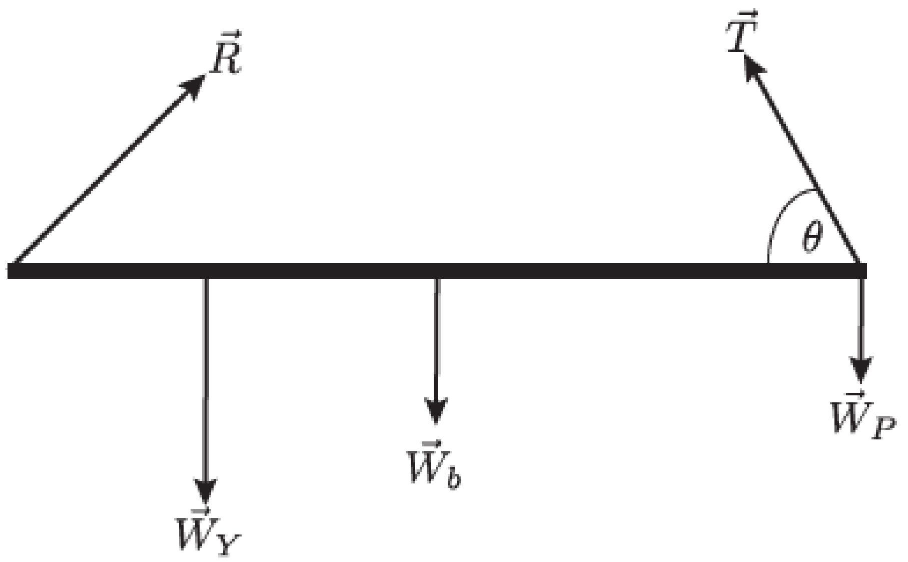
P2. A uniform horizontal beam with length $\ell = 8m$ and weight $W_b = 300N$ is attached to the wall by a pin connection. Its far end is supported by a cable that makes an angle of $\theta = 60°$ with the beam. Furthermore a picknick basket of weight $W_P = 100N$ is hanging from the far end of the beam. The hungry Yogy bear with a weight of $W_Y = 800N$ is standing on a beam at a distance $d = 2m$ from the wall.



(a) Calculate the magnitude of the tension in the cable.

(b) Calculate the force vector exerted by the pin on the beam.

(a) For the solution of this problem let us first draw the free body diagram for this problem.



So we can easily get the magnitude of the tension in the cable by considering the net torque of the system:

$$\sum \tau = -W_Y \cdot d - W_b \cdot \frac{\ell}{2} - W_P \cdot \ell + T \sin\theta \cdot \ell = 0$$

The only unknown is the magnitude of the tension in the cable, so we solve the equation for T:

$$T = \frac{W_Y \cdot d + W_b \cdot \frac{\ell}{2} + W_P \cdot \ell}{\sin\theta \cdot \ell} = \frac{800N \cdot 2m + 300N \cdot 4m + 100N \cdot 8m}{\sin 60^\circ \cdot 8m} = 519.6N$$

(b) In order to calculate the force vector exerted by the pin on the beam we have to consider the components of the net force:

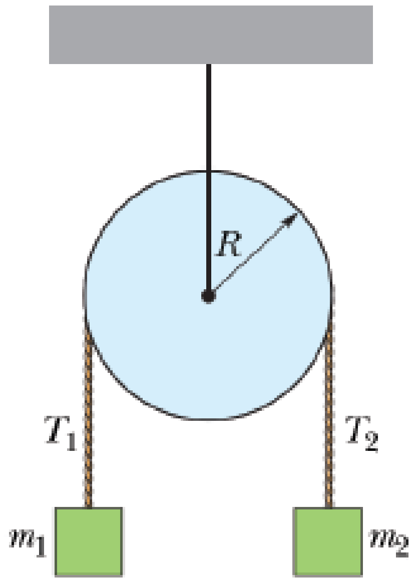
$$\begin{aligned}
\sum F_x &= R_x - T\cos\theta = 0 \Longrightarrow R_x = T\cos\theta = 519.6N\cos 60^\circ = 259.8N \\
\sum F_y &= R_y - W_Y - W_b - W_P + T\sin\theta = 0 \Longrightarrow \\
R_y &= W_Y + W_b + W_P - T\sin\theta = 800N + 300N + 100N - 519.6N\sin 60^\circ = 750N
\end{aligned}$$

Therefore the force vector becomes:

$$\vec{R} = 259.8N\hat{\imath} + 750N\hat{\jmath}$$

P3

A mass $m_1 = 0.46$ kg and a mass $m_2 = 0.5$ kg are connected over a massless cord passing over a pulley which is free to rotate, as shown in the figure below. The pulley has a radius of $R = 0.05m$. When released from rest, m_2 falls $0.75m$ in $5s$ without the cord slipping on the pulley.



(a) What is the magnitude of the linear acceleration of the masses and the angular acceleration of the pulley? (4P)

Solution:

As the acceleration is constant over the distance y and the time t, we can easily calculate the acceleration as

$$y = \frac{1}{2}at^2 \Longrightarrow a = \frac{2y}{t^2} = \frac{2 \cdot 0.75m}{(5s)^2} = 0.06\frac{m}{s^2}.$$

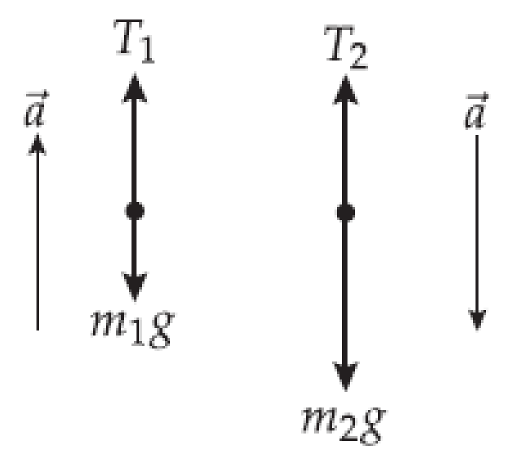
The angular acceleration is then:

$$\alpha = \frac{a}{R} = \frac{0.06\frac{m}{s^2}}{0.05m} = 1.2\frac{rad}{s^2}$$

(b) What are the magnitudes of the tensions T_1 and T_2? (4P)

Solution:

As the magnitude of the acceleration is already given, we can determine the magnitude of the tensions using the following free body diagrams for the masses m_1 and m_2:



From the Free Body Diagrams of m_1 and m_2 we get:

$$T_1 - m_1 g = m_1 a$$
$$T_1 = m_1(a+g)$$
$$T_1 = 0.46kg \cdot \left(0.06\frac{m}{s^2} + 9.8\frac{m}{s^2}\right) = 4.54N$$

$$T_2 - m_2 g = -m_2 a$$
$$T_2 = m_2(g-a)$$
$$T_2 = 0.5kg\left(9.8\frac{m}{s^2} - 0.06\frac{m}{s^2}\right) = 4.87N$$

(c) What is the pulleys moment of inertia? (2P)

Solution:

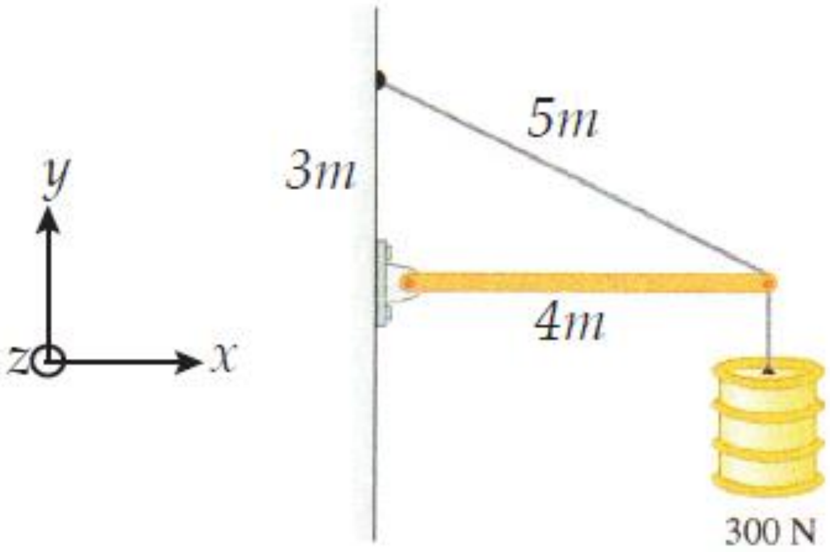
In order to calculate the moment of inertia, we have to consider the net torque:

$$
\begin{aligned}
T_1R - T_2R &= I\alpha \\
I &= \frac{T_1R - T_2R}{\alpha} \\
I &= \frac{(4.54N - 4.87N) \cdot 0.05m}{-1.2\frac{rad}{s^2}} = 0.014kg \cdot m^2
\end{aligned}
$$

$\alpha = -1.2\frac{rad}{s^2}$ because the rotation is clockwise.

P4

A uniform horizontal beam with a length of $l = 4.00$m and a weight of $W_b = 150$N is attached to a wall by a pin connection. Its far end is supported by a 5m long cable. Furthermore a weight of $W_A = 300$ N is attached at the far end of the beam.



(a) Find the magnitude of the tension in the cable. (4P)

Solution:
For the static equilibrium the net force and the net torque acting on the system has to be 0. For the calculation of the tension of the cable we just need to consider the net torque, as the angle of the tension is already given.

$$\sum \tau = -W_b \frac{l}{2} - W_A l + T \sin\theta l = 0$$

With $\sin\theta = 3/5$ we get for T

$$T = \frac{W_b/2 + W_A}{\sin\theta} = \frac{150N/2 + 300N}{3/5} = 625N$$

(b) Find magnitude and direction of the pin force exerted by the pin on the beam. (6P)

Solution:

For the calculation of the magnitude and direction of the pin force exerted by the pin on the beam, we have to consider the net force to be zero.

$$\begin{aligned} \sum F_x &= R\cos\phi - T\cos\theta = 0 \\ \sum F_y &= R\sin\phi + T\sin\theta - W_b - W_A = 0 \end{aligned}$$

$$\begin{aligned} R\cos\phi &= T\cos\theta && (1) \\ R\sin\phi &= -T\sin\theta + W_b + W_A && (2) \end{aligned}$$

Divide (2) by (1):

$$\tan\phi = \frac{-T\sin\theta + W_b + W_A}{T\cos\theta} = \frac{-625N\cdot\frac{3}{5} + 150N + 300N}{625N\cdot\frac{4}{5}} = 0.15 \Longrightarrow \phi = 8.54^\circ$$

Substitute $\phi = 8.54^\circ$ in (1) and get for the magnitude of the pin force:

$$R = \frac{T\cos\theta}{\cos\phi} = \frac{625N\cdot\frac{4}{5}}{\cos 8.53^\circ} = 505.6N$$