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# Uninformed Hawking radiation

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**Abstract** – We show in detail that the Parikh-Wilczek tunneling method (PWTM), which was designed for resolving the information loss problem in Hawking radiation (HR) fails whenever the radiation occurs from an isothermal process. The PWTM aims to produce a non-thermal HR which adumbrates the resolution of the problem of unitarity in quantum mechanics (QM), and consequently the entropy (or information) conservation problem. The effectiveness of the method has been satisfactorily tested on numerous black holes (BHs). However, it has been shown that the isothermal HR, which results from the emission of the uncharged particles of the linear dilaton BH (LDBH) described in the Einstein-Maxwell-Dilaton (EMD) theory, the PWTM has vulnerability in having non-thermal radiation. In particular, we consider Painlevé-Gullstrand coordinates (PGCs) and isotropic coordinates (ICs) in order to prove the aforementioned failure in the PWTM. While carrying out calculations in the ICs, we also highlight the effect of the refractive index on the null geodesics.

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**Introduction.** – As is well known, Hawking [1] theoretically proved that BHs could emit radiation (often called HR), which implies that a BH would eventually evaporate away, leaving nothing over time. According to the principles of QM, complete information about a system is encoded in its wave function. The evolution of the wave function is determined by a unitary operator, and unitarity implies that information is conserved in the quantum sense. However, the combination of QM and general relativity suggests that physical information could permanently disappear in a BH, allowing many physical (pure quantum) states to devolve into the mixed state of HR. This phenomenon is called the BH information loss paradox (the reader may refer to [2] for the topical review), which underscores the apparent violation of unitarity in the process of HR. Among the many attempts at a resolution of this problem, the most promising one came at the turn of this century, belongs to Parikh and Wilczek (PW) [3]. The theorem states that when a virtual pair is created just inside the BH horizon, the positive-energy particle (real particle) can tunnel out the BH horizon by a process similar to the QM tunneling, whereas the negative-energy particle (antiparticle) continues to stay in the BH. Conversely, as one would expect from particle-antiparticle symmetry, if a virtual pair is created just outside the horizon, the antiparticle can tunnel inward, while the real particle will eventually escape to spatial infinity.

In the PWTM, the conservation of energy is enforced. Therefore, the mass of the BH must continuously decrease while it radiates. Besides this, the information-carrying particle is modelled as a thin spherical shell with energy  $\omega$ . Those shells could tunnel through the potential barrier, following the principles of QM. In short, the whole tunneling process is considered semiclassically, and the transmission coefficient is determined by the classical action of the particle with the aid of the Wentzel-Kramers-Brillouin (WKB) method [4]. As a result, the obtained spectrum is not precisely thermal, and this also leads to the unitarity of the underlying quantum theory and the conservation of information [5]. On the other hand, so far, the solution of the PWTM to the problem of the information paradox has not convinced everyone, and hence it has also remained debatable (one can see the extensive review on the PWTM analysis [6] and references therein). Furthermore, the PWTM has also extended to the HR analysis of the non-asymptotically flat (NAF) BHs (see for instance [7–9]).

The PWTM through the quantum horizon of a LDBH geometry, which is the solution to the EMD theory [10–12], and its extended theories [13], was studied in [9,14,15]. This BH is a NAF, four dimensional, spherically symmetric and static dilatonic spacetime. It was shown by [14,15] that in the proposed PW setup, there is no correlation between different subsequently emitted particles, which

reflects the fact that information does not come out continuously during the evaporation process. Then some possible scenarios to conserve the information were given in [14]. To this end, the back reaction effects were taken into account. However, we believe that, in those studies [14,15], the main point that the PWTM does not yield non-thermal radiation for a BH evaporating isothermally, has not been stressed enough. Therefore, the fundamental motivation of the present study is to highlight that the PWTM can not be the general procedure for having non-thermal HR.

In this paper, in addition to the PGCs, we also employ the PWTM within the ICs that has not been studied before for the LDBHs. In particular, in the IC system we represent in detail how the Hawking temperature can be precisely obtained within the framework of the PWTM, and how the PWTM is ineffective in achieving non-thermal radiation.

**Pure thermal radiation of the LDBH.** – The action of the EMD in 3 + 1 dimensions (4D) is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - 2\partial_\mu \phi \partial^\mu \phi - e^{-2\beta\phi} F^2), \quad (1)$$

where  $\phi$  is the dilatonic field with a coupling constant  $\beta$  and  $F^2 = F_{\mu\nu} F^{\mu\nu}$  in which  $F_{\mu\nu}$  is the electromagnetic field or the  $U(1)$  gauge field. Static, spherically symmetric NAF solutions in 4D were obtained in [16]. Among them the LDBH [10], which corresponds to the case of  $\beta = 1$ , is given by

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2)$$

where the metric functions and the fields are given by

$$f = r_0^{-1}(r - b), \quad R^2 = rr_0, \quad (3)$$

$$e^{2\phi} = \frac{r}{r_0}, \quad rt = \frac{Q}{r_0^2}, \quad (4)$$

in which  $b$  represents the event horizon  $r_h$ , which is also related to the mass. In general, the mass of a NAF BH is computed via the Brown-York quasilocal mass definition [17]. Thus, one can compute the quasilocal mass of the LDBH as

$$M = \frac{b}{4}. \quad (5)$$

Furthermore, the another parameter  $r_0$  is related with the charge  $Q$  of the LDBH through

$$r_0 = \sqrt{2}Q. \quad (6)$$

It is worth noting that both  $b$  and  $r_0$  parameters have the same dimension in the geometrized unit system [18] since the mass and the charge are represented by the  $[L]$  geometrical dimension. The conventional definition of the Hawking temperature  $T_H$  [18] is formulated in terms of

the surface gravity  $\kappa$  as  $T_H = \frac{\kappa}{2\pi}$ . For the metric (2),  $T_H$  becomes

$$T_H = \frac{\kappa}{2\pi} = \left. \frac{\partial_r f}{4\pi} \right|_{r=r_h}, \quad (7)$$

which yields

$$T_H = \frac{1}{4\pi r_0}. \quad (8)$$

It is clear that the obtained temperature is independent of mass, and consequently it is constant. Therefore,  $\Delta T_H = 0$ , which means that the radiation is an isothermal process. Thus, HR of the LDBH is such a special radiation that the energy transfer out of it happens at a particular slow rate so that thermal equilibrium is always satisfied. Furthermore, the extreme LDBH ( $M = b = 0$ ) is still a BH and possesses a clashed *singularity-pointlike horizon* structure. Its singularity is null, and the delivery time for an emitted signal from the horizon to an external observer is infinite [11,12]. This extreme BH can be used to describe the LDBH remnant [14]. Using the massless Klein-Gordon equation, it is proven that such a remnant cannot radiate, as expected, and its Hawking temperature is zero. By taking the tunneling formalism with subsequent emissions and quantum gravity corrected entropy into consideration, Sakalli *et al.* [14] also showed that the entropy of the extreme LDBH can be derived.

In order to employ the PWTM and investigate the Hawking temperature of the LDBH, one should choose a suitable coordinate system which is not singular at the event horizon. Along the line of PW [3], we firstly consider the PGCs [19,20] by applying the following coordinate transformation:

$$dT = dt + \frac{\sqrt{1-f}}{f} dr, \quad (9)$$

where the coordinate  $T$  denotes the time in the PGCs, which measures the proper time. Thus the line-element (1) transforms into

$$ds^2 = -f dT^2 + 2\sqrt{1-f} dT dr + dr^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (10)$$

At  $r = r_h$  (*i.e.*,  $f = 0$ ), the metric coefficients are all regular, and indeed the coordinates are all well behaved there. Since we think the particle as a spherical shell, during the tunneling process, the particle does not have motion in  $(\theta, \varphi)$ -directions. Thus, the radial null geodesics can be obtained as

$$\dot{r} = \frac{dr}{dT} = \pm 1 - \sqrt{1 - r_0^{-1}(r - 4M)}, \quad (11)$$

by which the upper (lower) sign corresponds to the outgoing (ingoing) geodesics. In [21], it was shown that the ratio of emission and absorption probabilities for energy  $E$  is

$$\frac{P_{emission}}{P_{absorption}} = e^{-\frac{E}{T_H}}. \quad (12)$$

In the WKB approximation [4], these probabilities are related to the outgoing/ingoing imaginary part of the

particle's action ( $\text{Im}S_{out}/\text{Im}S_{in}$ ) as follows:

$$P_{emission} = e^{-2\text{Im}S_{out}}, \quad P_{absorption} = e^{-2\text{Im}S_{in}}. \quad (13)$$

Since the tunnelling ratio is expressed as

$$\Gamma = \frac{P_{emission}}{P_{absorption}} = e^{-\frac{E}{T_H}} = e^{-2\text{Im}S}, \quad (14)$$

where  $\text{Im}S$  denotes the net imaginary part of particle's action [22], we have

$$\text{Im}S = \text{Im}S_{out} - \text{Im}S_{in}. \quad (15)$$

Moreover, the imaginary part of the action for the ingoing particle is given by

$$\text{Im}S_{in} = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{P_r} dp'_r dr, \quad (16)$$

where  $p_r$  denotes the canonical momentum along the  $r$ -direction [3].  $r_{in}$  and  $r_{out}$  represent the radial distance of the event horizon before and after the HR, respectively. Since the BH shrinks in the process of the HR,  $r_{in} > r_{out}$ . According to the PWTM, we should fix the total mass of the system ( $M$ ) and allow the BH to fluctuate. Also, we consider the chargeless particle as a thin spherical shell of energy  $\omega$ . After taking into account the self-gravitational effect, the mass of the BH decreases as  $M \rightarrow M - \omega$ . Furthermore, Hamilton's equation  $\dot{r} = \frac{dH}{dp_r}$  can be used to transform variables from momentum to energy. Thus eq. (16) becomes

$$\text{Im}S_{in} = \text{Im} \int_{r_{in}}^{r_{out}} \int_M^{M-\omega} \frac{dr}{\dot{r}} dH. \quad (17)$$

Then, we can switch integration variables from  $H$  to the particle's energy  $\omega$ . Letting  $H = M - \omega'$ , we consequently get  $dH = -d\omega'$ . So, we have

$$\begin{aligned} \text{Im}S_{in} &= \text{Im} \int_{r_{in}}^{r_{out}} \int_0^\omega \frac{dr}{\dot{r}} (-d\omega'), \\ &= \text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{dr}{1 + \sqrt{1 - r_0^{-1} [r - 4(M - \omega')]} } (d\omega'), \\ &= \text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{\Psi_{in}}{r - 4(M - \omega')} dr (d\omega'), \end{aligned} \quad (18)$$

where

$$\Psi_{in} = r_0 - \sqrt{r_0 [r_0 - r + 4(M - \omega')]}]. \quad (19)$$

From eq. (18), one can see that there is a contour integral in the complexified  $r$ -plane picks up a residue at  $r = 4(M - \omega')$ . After deforming the contour around the pole (pushing the pole into the upper half complex  $r$ -plane), we get a prefactor of  $-i\pi$ . For the detailed description of residue calculus, one may refer to [23]. Evaluating the integral, we obtain

$$\text{Im}S_{in} = 0. \quad (20)$$

If we repeat the same procedure for the imaginary part of the action for the outgoing particle, we have

$$\text{Im}S_{out} = -\text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{\Psi_{out}}{r - 4(M - \omega')} dr d\omega', \quad (21)$$

in which

$$\Psi_{out} = r_0 + \sqrt{r_0 [r_0 - r + 4(M - \omega')]}], \quad (22)$$

$r$ -integral seen in eq. (21), has also a single pole at  $r = 4(M - \omega')$ . Therefore, one can get

$$\text{Im}S_{out} = \text{Im}S = 2\pi\omega r_0. \quad (23)$$

The tunneling rate (14) for a particle outwards through the horizon thus turns out to be

$$\Gamma = \exp(-4\pi\omega r_0). \quad (24)$$

So the obtained temperature

$$T = \frac{1}{4\pi r_0}, \quad (25)$$

is nothing but the standard Hawking temperature given in eq. (8). However there is an intriguing issue in this result: Although the energy conservation is enforced, the spectrum of the radiation is still precisely thermal. According to our knowledge, this (isothermal HR) is a unique case for the PWTM that it could not modify the pure thermal character of the HR.

Now, we want to verify our result in another regular coordinate system. For this purpose, we consider the LDBH within the IC system. The ICs have several interesting features similar to the PGCs: The time direction is a Killing vector and Landau's condition of the coordinate clock synchronization [24] is automatically satisfied. The LDBH spacetime in the ICs has been recently studied by Sakalli and Mirekhtariy [25]. By following the associated transformation given in that reference,

$$r = \frac{1}{4\rho} (\rho + b)^2, \quad (26)$$

we can express the LDBH metric in the ICs as

$$ds^2 = -F dt^2 + G [d\rho^2 + \rho^2 (d\theta^2 + \sin^2 \theta d\varphi^2)], \quad (27)$$

with

$$F = \frac{1}{4\rho r_0} (\rho - b)^2, \quad G = \frac{r_0}{4\rho^3} (\rho + b)^2. \quad (28)$$

Moreover, the event horizon in the IC is located at  $\rho_h = b$ . From metric (27), one can obtain the radial null geodesics as

$$\dot{\rho} = \frac{d\rho}{dt} = \pm \sqrt{\frac{F}{G}} = \pm \frac{1}{n}, = \pm \frac{\rho(\rho - b)}{r_0(\rho + b)}, \quad (29)$$

where  $n$  is the refractive index of the medium of the LDBH geometry [25] and it is the deterministic parameter on the imaginary part of the action for an outgoing (tunneling)

particle:

$$\begin{aligned} \text{Im}S_{out} &= \text{Im} \int_{\rho_{in}}^{\rho_{out}} \int_0^\omega n d\rho(-d\omega'), \\ &= -r_0 \text{Im} \int_0^\omega \int_{z_{in}}^{z_{out}} \frac{[\rho + 4(M - \omega')]}{[\rho - 4(M - \omega')]} \frac{d\rho}{\rho} (d\omega'). \end{aligned} \quad (30)$$

The  $\rho$ -integral has a pole at  $4(M - \omega')$ . However, one must be cautious about a subtle point, which was pointed out in [25–27] that when one deforms the contour of the integral around the pole, the semicircular contour in eq. (30) gets transformed into a quarter circle. Namely, we obtain a prefactor of  $-i\pi/2$  rather than  $-i\pi$ . Thus

$$\text{Im}S_{out} = \pi\omega r_0. \quad (31)$$

Similarly, we can obtain the imaginary part of action for the ingoing particles as

$$\text{Im}S_{in} = -\pi\omega r_0, \quad (32)$$

so that from eq. (15) we have

$$\text{Im}S = 2\pi\omega r_0. \quad (33)$$

This result is in agreement with eq. (23), and it leads to the conventional Hawking temperature (8). In short, the failure of the PWTM in revealing non-thermal radiation is proven also in the ICs.

**Conclusion.** – In this article, it has been shown that the original PWTM method cannot convert isothermal HR to a non-thermal radiation. In particular, we have used the LDBH, which radiates isothermally. In order to use the PWTM, the PGCs and ICs, which are two well-behaved coordinate systems, have been chosen. In both coordinate systems, it has been straightforwardly shown that, in spite of the energy conservation being taken into account, the pure thermal character of the HR does not modify. Namely, the original PWTM does not resolve the information loss paradox in the LDBH spacetime. Hence, it is our belief that seeking an alternative model to the PWTM, beside the work of [14], which can produce the non-thermal radiation from the LDBH will be useful in the information theory. This is going to be our next problem in the near future.

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