Weak and Strong Warm Logamediate Anisotropic Inflationary Universe Model

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(Dated: January 1, 2020)

In this paper we investigate a warm inflation scenario of a locally rotationally symmetric Bianchi I model using a background of modified Chaplygin gas. We determine the field equations and perturbations parameters, such as; the scalar power spectrum, scalar spectral index, scalar potential and tensor to scalar ratio under slow roll approximation. We determine these parameters in a directional of Hubble parameter during the both weak and strong logamediate inflationary regimes. These cosmological parameters show that the anisotropic model is compatible with WMAP 7 from the 2018 Planck observational data.

PACS numbers:
Keywords: warm inflation; slow roll approximation; scalar field.

I. INTRODUCTION

The standard universe model (hot big-bang cosmology) successfully explains the observations of a cosmic microwave background (CMB) but there are still some unresolved issues regarding the origins of the fluctuations, horizon, flatness and magnetic monopole. Inflation is successfully resolved in current theory, and is a paradigm for addressing the shortcomings of standard model issues [1–8]. Scalar field, as a primary ingredient of inflation, provides the causal interpretation of the origin of large scale structure (LSS) distribution and observed anisotropy of CMB [9, 10]. Inflationary standard models are classified into slow-roll and reheating epochs. In a slow-roll period, potential energy dominates kinetic energy. All interactions between scalar (inflatons) and other fields are neglected: hence the universe inflates [11]. Subsequently, the universe enters into a reheating period where the kinetic energy is comparable to the potential energy. Thus, the inflaton starts an oscillation at about the minimum of its potential, losing its energy to other fields that are present in the theory [12]. After this epoch, the universe is filled with radiation. According to the current universe model, cold inflation, rather than warm inflation is the ending stage of the inflating universe [13, 14]. Warm inflation is the only way for thermal radiation production in the reheating epoch. The formation of LSS and also formation of initial fluctuation can be produced by the thermal fluctuations with the constant density which can be the affects of dissipation. The Hubble parameter should be less than decay rate, according to the process of microscope the thermal particles can be produced. The radiation dominated phase is easily entered into by the universe, when the inflating era can be stopped. Finally, the remaining matter particles are produced [15, 16]. Many aspect of warm inflation are discussed in the literature [17]. The motivation of warm inflation is completely different from their to its result. In the inflation era, the dissipative effects can lead to a friction term in the equation of motion, and can also describe the dissipative coefficient. The cases of low, high and constant temperature regimes are described in the dissipation coefficient [18–29]. The dissipation coefficient is discuss in regards to the two cases, weak $R << 1$ and strong $R >> 1$ [30, 31]. In the scenario of a warm inflation era, the general dissipation coefficient can be written as;

$$\Gamma(T, \varphi) = C_\varphi \frac{T^{\tilde{z}}}{\varphi^{\tilde{z}-1}}$$

where T is the temperature of the thermal bath,φ is the scalar field, C_φ is a dissipation microscopic dynamics and z is an integer term for the different specific values s.t $z = 3, 1, 0, -1$ for low, high, and constant temperature. The

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value of $z = 1$ represents high temperature (SUSY-case) $\Gamma \propto T$, and $z = 0$ leads to normal temperature (exponentially decaying propagator in the SUSY case) $\Gamma \propto \varphi$ and for $z = -1$ non SUSY case leads to decay rates is $\Gamma \propto \frac{\varphi^2}{T}$ and for $z = 3$ the most common form $\Gamma \propto \frac{T^3}{\varphi^2}$leads to a most common form for considering logamediate models [32]-[35] According to the conditions of warm inflation, it can be existence of thermal radiation and temperature $T >> H$. The thermal fluctuations is proportional to T and H. According to Chaplygin gas with an exotic equation of state and with negative pressure can be described by

$$\rho_{cg} = -\frac{\tilde{\chi}}{\rho_{cg}},$$

and

$$\rho_{gcg} = -\frac{\tilde{\chi}}{\rho_{gcg}^{\lambda}}.$$

This equation can be extended in the form of generalized Chaplygin gas and $-\tilde{\chi}$, and λ is a constant parameter. For the value of $\lambda = 1$, it is converted to the original Chaplygin gas. However, the Chaplygin gas is also modified the equation of state,

$$p_{mcg} = \tilde{\zeta}\rho_{mcg} - \frac{\tilde{\chi}}{\rho_{mcg}^{\lambda}}. \tag{1}$$

In Section 2 of this paper, we discuss the basic formalism of warm inflation in the view of MCG. In Section 3 and 4, we discuss the weak and strong regime in the MCCG. Here, we also determine the explicit expressions of inflation rate and decay rate, as well as the perturbation parameters: scale factor, tensor to scalar ratio, and scalar power spectrum spectral index. We then discuss the graphical behavior in the context of the 2018 Planck observational data. In Section 5 we summarize the results.

II. MODIFIED CHAPLYGIN GAS INSPIRED INFLATION

In this section, we discuss the general form of modified Chaplygin gas in the view of the dissipative coefficient for the inflaton decay rate Γ, and we have formalism of equation of state in scenario of MCCG;

$$p_{mcg} = \tilde{\zeta}\rho_{mcg} - \frac{\tilde{\chi}}{\rho_{mcg}^{\lambda}}, \tag{2}$$

where $\tilde{\zeta}$ and $\tilde{\chi}$ are two constant parameters and $0 \leq \lambda \leq 1$. Also P_{mcg} represent the pressure and ρ_{mcg} is the energy density of Chaplygin gas. We find the energy density of Chaplygin gas from the equation of stress energy equation, the scale factor a,

$$\begin{aligned} \rho_{mcg} &= \left(\frac{\tilde{\chi}}{\tilde{\zeta}+1} + \frac{\tilde{\tilde{\xi}}}{\varsigma^{(\tilde{m}+2)(\lambda+1)(\tilde{\zeta}+1)}}\right)^{\frac{1}{\lambda+1}} \\ &= \rho_{mcg0}\left(\tilde{\chi}_s + \frac{1-\tilde{\chi}_s}{\varsigma^{(\tilde{m}+2)(\tilde{\zeta}+1)(\lambda+1)}}\right)^{\frac{1}{\lambda+1}}, \end{aligned} \tag{3}$$

where $\tilde{\chi}_s = \frac{\tilde{\chi}}{\tilde{\zeta}+1}\frac{1}{\rho_{mcgo}^{\lambda+1}}$. According to Equation (2.2), we introduce some parameters: $\tilde{\chi}_s$, and $\tilde{\zeta}$. Here, λ and $\tilde{\tilde{\xi}}$ are positive integration constants. We use the differential age of old galaxies such that the oscillation peak parameter, Baryonic acoustic, SN Ia data and growth index for the different and specific values (for best-fit) are obtained by $\tilde{\chi}_s = 0.8252$, $\tilde{\zeta} = 0.0046$ and $\lambda = 0.1905$. These contribute to the two equations, energy density of matter $\rho_{\tilde{m}}$ and energy density of the radiation field ρ_{φ} in the background of inflation:

$$\left(\frac{\tilde{\chi}}{\tilde{\zeta}+1} + \rho_{\tilde{m}}^{(\lambda+\lambda\tilde{\zeta}+\tilde{\zeta}+1)}\right)^{\frac{1}{\lambda+1}} \rightarrow \left(\frac{\tilde{\chi}}{\tilde{\zeta}+1} + \rho_{\psi}^{(\lambda+\lambda\tilde{\zeta}+1+\tilde{\zeta})}\right)^{\frac{1}{\lambda+1}}. \tag{4}$$

Consider a flat universe: then, the radiation field and inflation field φ are self interacting. We then would write the Friedmann equation in the form:

$$H_2^2 = \frac{\kappa}{1+2\tilde{m}}\left(\left[\frac{\tilde{\chi}}{1+\tilde{\zeta}} + \rho_\varphi^{(1+\lambda)(\tilde{\zeta}+1)}\right]^{\frac{1}{\lambda+1}} + \rho_\gamma\right), \tag{5}$$

where H is the Hubble parameter, defined $H = \frac{\dot{a}}{a}$, and $\kappa = 8\pi G$. According to modified Friedmann equation, we suppose that the inflation field φ and the radiation field in the scenario of flat universe yield:

$$\dot{\rho_\varphi} + (\tilde{m}+2)(\rho_\varphi + P_\varphi) = -\Gamma\dot{\varphi}^2 \Rightarrow \ddot{\varphi} + (\tilde{m}+2)\dot{\varphi} + \acute{V} = -\Gamma\dot{\varphi}^2, \tag{6}$$

and

$$\dot{\rho}_\gamma + \frac{4}{3}(\tilde{m}+2)H_2\rho_\gamma = \Gamma\dot{\varphi}^2, \tag{7}$$

$$\tilde{\rho_\varphi} = \frac{\dot{\varphi}^2}{2} + V(\varphi) \tag{8}$$

$$\tilde{P_\varphi} = \frac{\dot{\varphi}^2}{2} - V(\varphi) \tag{9}$$

Here ρ_φ is the energy density and P_φ is the pressure. Both functions are related to the same field. The similarly term is, consequently, $V(\varphi)$ a scalar potential. The condition of the energy density of the radiation field, if $\rho_\varphi >> \rho_\gamma$ then Eq.(2.4)

$$\begin{aligned} H_2^2 &\approx \frac{\kappa}{1+2\tilde{m}}\left(\left[\frac{\tilde{\chi}}{1+\tilde{\zeta}} + \rho_\varphi^{(1+\lambda+\lambda\tilde{\zeta}+\tilde{\zeta})}\right]^{\frac{1}{1+\lambda}}\right) \\ &= \frac{\kappa}{2\tilde{m}+1}\left[\frac{\tilde{\chi}}{\tilde{\zeta}+1} + \left(V(\varphi) + \frac{\dot{\varphi}^2}{2}\right)^{(\lambda+\lambda\tilde{\zeta}+1+\tilde{\zeta})}\right]^{\frac{1}{1+\lambda}}. \end{aligned} \tag{10}$$

By solving these equations in terms of the field using Eqs.(2.5) and (2.9),

$$\begin{aligned} \dot{\varphi}^2 &= \frac{(2+4\tilde{m})(-\dot{H}_2)}{\kappa(\tilde{m}+2\tilde{\zeta}+\tilde{m}\tilde{\zeta}+2)(1+R)}\left[\frac{(1+2\tilde{m})H_2^2}{\kappa}\right]^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \\ &\times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{\left(\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\tilde{\zeta}}{(\tilde{\zeta}+1+\lambda\tilde{\zeta}+\lambda)}\right)}, \end{aligned} \tag{11}$$

we characterized a new parameter as follow:

$$R \equiv \frac{\Gamma}{(\tilde{m}+2)(H_2)}. \tag{12}$$

According to this condition $\dot{\rho_\gamma} << \frac{4}{3}(\tilde{m}+2)H_2\rho_\gamma$ by combining Eq. (2.6) and (2.10),

$$\begin{aligned} \tilde{\rho_\gamma} &= \frac{3\Gamma\dot{\varphi}^2}{4(\tilde{m}+2)H_2} = \frac{3\Gamma(-\dot{H}_2)}{2\kappa(H_2)(1+\tilde{\zeta})(1+R)} \\ &\times \left[\frac{(2\tilde{m}+1)H_2^2}{\kappa}\right]^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}}\left[\frac{(2\tilde{m}+1)}{(\tilde{m}+2)^2}\right] \\ &\times \left[1 - \frac{\tilde{\chi}}{\tilde{\zeta}+1}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(1+\lambda)}\right]^{\left(\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\tilde{\zeta}}{(\tilde{\zeta}+1+\lambda\tilde{\zeta}+\lambda)}\right)}. \end{aligned} \tag{13}$$

The thermalized energy density is $\rho_\gamma = C_\gamma T^4$. According to the Minimal Super Symmetric Standard model (MSSM), $C_\gamma = \frac{\Pi^2 g_*}{30}$, $g_* = 228.75$ and $C_\gamma = 70$ [36], by solving Eq.(2.12):

$$T = \left[\frac{3\Gamma(-\dot{H}_2)(1+2\tilde{m})}{2\kappa H_2 C_\gamma(\tilde{m}+2)^2(1+\tilde{\zeta})(1+R)}\right]^{\frac{1}{4}} \left[\frac{(1+2\tilde{m})H_2^2}{\kappa}\right]^{\frac{-\tilde{\zeta}}{(4\tilde{\zeta}+4)}} \times \left[1-\frac{\tilde{\chi}}{\tilde{\zeta}+1}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{\frac{-\tilde{\zeta}-\lambda\tilde{\zeta}-\lambda}{(4+4\tilde{\zeta}+4\lambda\tilde{\zeta}+4\lambda)}}, \tag{14}$$

By considering Eqs. (2.9), (2.10), and (2.13),

$$V = \left(\frac{(1+2\tilde{m})H_2^2}{\kappa} - \frac{\tilde{\chi}}{1+\tilde{\zeta}}\right)^{\frac{1}{(1+\tilde{\zeta}+\lambda+\lambda\tilde{\zeta})}} + \frac{\dot{H}_2}{\kappa(1+\tilde{\zeta})(1+R)} \times \frac{(1+2\tilde{m})}{(\tilde{m}+2)}\left[\frac{(1+2\tilde{m})(H_2^2)}{\kappa}\right]^{\frac{-\tilde{\zeta}}{\tilde{\zeta}+1}} \times \left[1-\frac{\tilde{\chi}}{\tilde{\zeta}+1}\left(\frac{(2\tilde{m}+1)H_2^2}{\kappa}\right)^{-(1+\lambda)}\right]^{-\frac{(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(1+\lambda)}}, \tag{15}$$

The dissipative coefficient can be obtained as,

$$\Gamma^{\frac{4-z}{4}} = \left[\frac{3(1+2\tilde{m})(-\dot{H}_2)}{2\kappa(C_\gamma)H_2(\tilde{m}+2)^2(1+\tilde{\zeta})(1+R)}\right]^{\frac{z}{4}} \left[\frac{(1+2\tilde{m})(H_2^2)}{\kappa}\right]^{\frac{-z\tilde{\zeta}}{4(1+\tilde{\zeta})}} \times C_\varphi \varphi^{1-z}\left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{-\frac{z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4+4\lambda)}}, \tag{16}$$

In this era, the scale factor of logamediate inflationary model is given by,

$$a(t) = \exp^{f(lnt)^g} f > 0 and g > 1 \tag{17}$$

where g and f are two dimensionless constant parameter [37]. The next three sections, we will discuss the two cases weak dissipative regime and strong dissipative regime.

III. THE WEAK DISSIPATIVE REGIME

In this case, the weak $(R << 1)$ function can be converted in number of scalar field's φ , by using Eq. (2.10) and (2.16), we get

$$\varphi(t) - \varphi_o = \frac{\tilde{\tau}[t]}{\tilde{\omega}}, \tag{18}$$

Where φ_o is a constant term of this integration , and $\tilde{\omega}$ is a constant term and $\tau\tilde{[}t]$ is a function of comic time is given by, (with condition $\varphi_0 = 0$)

$$\tilde{\omega} = \sqrt{2}(1+\tilde{\zeta})^{\frac{3}{2}}\sqrt{(\tilde{m}+2)}(\frac{1}{f*g})^{\frac{1-\tilde{\zeta}}{2(1+\tilde{\zeta})}}(\frac{\kappa}{1+2\tilde{m}})^{\frac{1}{2(1+\tilde{\zeta})}}$$

$$\begin{aligned}\tau[t] &= \frac{2(1+\tilde{\zeta})^2 Gamma\left[\frac{1+3\tilde{\zeta}+g-\tilde{\zeta}g}{2(1+\tilde{\zeta})}, \frac{-\tilde{\zeta}Log[t]}{1+\tilde{\zeta}}\right](ln[t])^{\frac{(1-\tilde{\zeta})(1-g)}{2(1+\tilde{\zeta})}}}{\tilde{\zeta}} \\ &\times \left(\frac{-\tilde{\chi}t^{\frac{-2(1+\tilde{\zeta})}{(1+\tilde{\zeta})(1+\lambda)}}(\tilde{\zeta}+\lambda+\tilde{\zeta}\lambda)}{(1+\tilde{\zeta})(1+\lambda)}\right) \\ &\times Gamma[1-\frac{(-1+g)(3+4\lambda+\tilde{\zeta}(5+4\lambda))}{2(1+\tilde{\zeta})}, \\ &- \frac{(2+3\tilde{\zeta}+2\lambda+2\tilde{\zeta}\lambda)Log[t]}{2(1+\tilde{\zeta})}](ln[t])^{\frac{1+3\tilde{\zeta}+g-\tilde{\zeta}g}{2+2\tilde{\zeta}}} \\ &\times \left(\frac{-(2+3\tilde{\zeta}+2\lambda+2\tilde{\zeta}\lambda)Log[t]}{1+\tilde{\zeta}}\right)^{-1+\frac{(-1+g)(3+4\lambda+\tilde{\zeta}(5+4\lambda))}{2(1+\tilde{\zeta})}}\end{aligned}$$

Under the slow slow roll approximation, we formulate the Gamma function and also we find out the scalar potential in terms of scalar field φ by using $\frac{\dot{\varphi}^2}{2} < V(\varphi)$, we get

$$V(\varphi) \approx \left[\left(\frac{f^2(2\tilde{m}+1)g^2}{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^2(ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{(2-2g)}}\right)^{1+\lambda} - \frac{\tilde{\chi}}{\tilde{\zeta}+1}\right]^{\frac{1}{(1+\tilde{\zeta})(1+\lambda)}}, \quad (19)$$

we can written as constant dissipative coefficient in terms of φ as follows,

$$\begin{aligned}\Gamma(\varphi) &= \left[\frac{3(1+2\tilde{m})}{\kappa(C_r)(2+2\tilde{\zeta})(\tilde{m}+2)^2(ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))}\right]^{\frac{z}{4-z}} \\ &\times \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^2(ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{2(1-g)}}\right)^{\frac{-z\tilde{\zeta}}{(1+\tilde{\zeta})(4-z)}} C_\varphi^{\frac{4}{4-z}}\varphi^{\frac{4(1-z)}{4-z}} \\ &\times \left[1-\frac{\tilde{\chi}}{1+\varpi}\left(\frac{\kappa((\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^2(ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi]))^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{-z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(1+\lambda)(4-z)}}, \quad (20)\end{aligned}$$

In a cosmological time, the number of e-folds N is interpolated between two different time initial t_1 and final t_2 as follows,

$$\begin{aligned}N &= \left(\frac{\tilde{m}+2}{3}\right)\int_{t_1}^{t_2} H_2 dt \\ &= \frac{f(\tilde{m}+2)}{3}\left[(ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2]))^g - (ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_1]))^g\right], \quad (21)\end{aligned}$$

According to inflationary scenario, anisotropic model can be proposed by [38, 39]. Slow roll parameter determine's the degree of the anisotropy. Anisotropy during inflation cannot be completed neglected because slow roll parameter is factual known as order of a percent. Dimensionless slow roll parameter can be expressed ϵ and η by [40]. These parameters presented in function of φ defined as,

$$\epsilon = -\left(\frac{3}{\tilde{m}+2}\right)\frac{\dot{H}_2}{H_2^2}, \quad (22)$$

and

$$\eta = -\left(\frac{3}{\tilde{m}+2}\right)\frac{\ddot{H}_2}{\dot{H}_2 H_2}, \quad (23)$$

In inflationary scenario in terms of scalar filed slow roll parameter, it is a nearest of early and possible stage of $\epsilon = 1$,

$$\varphi_1 = \frac{1}{\varpi}\left[\tau\exp\left(\frac{3}{\tilde{m}+2}\frac{1}{fg}\right)^{\frac{1}{g-1}}\right], \quad (24)$$

There are four ways in which scalar perturbations can be represented i.e scalar spectral indices and scalar tensor power spectra n_s, n_T and $P_R[k], P_T[k]$ According to standard scalar field, the scalar density perturbations can be written as in the form of of $P_R^{\frac{1}{2}} = (\frac{\tilde{m}+2}{3})\frac{H_2}{\varphi}\delta\varphi$ is derive by $\delta\varphi^2 \simeq (\frac{m+2}{3})H_2 T$ [41]-[43]. The scalar power spectrum is obtained by Eqs. (2.10) ,(2.13) and (2.15)

$$P_R = F_1 \varphi^{\frac{1-z}{4-z}} t^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} (ln[t])^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(z-3)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (25)$$

This function is converted in the form of scalar field φ,

$$P_R = F_1 \varphi^{\frac{1-z}{4-z}} (\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2])^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}} \times (ln(\tilde{\tau}^{-1}[\tilde{\omega}\varphi_2]))^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(1+\tilde{\zeta})(z-3)}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (26)$$

where F_1 is a constant term then,

$$F_1 = (\frac{m\tilde{+}2}{3})^3 \left(\frac{\kappa(1+\tilde{\zeta})(\tilde{m}+2)}{2(1+2\tilde{m})}\right)\left(\frac{3(1+2\tilde{m})C_\varphi}{2\kappa C_\gamma(\tilde{m}+2)^2(1+\tilde{\zeta})}\right)^{\frac{1}{4-z}} \times \left[\frac{1+2\tilde{m}}{\kappa}\right]^{\frac{(3-z)\tilde{\zeta}}{(4-z)(1+\tilde{\zeta})}} (fg)^{\frac{(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)+(z-3)(1+\tilde{\zeta})}{(1+\tilde{\zeta})(4-z)}},$$

$$P_R = F_2(\tilde{\tau}(G(N)))^{\frac{1-z}{4-z}} (G[N])^{\frac{(3z-11)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+2(3-z)}{(1+\tilde{\zeta})(4-z)}} \times (ln(G[N]))^{\frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)]+(g-1)(z-3)}{(1+\tilde{\zeta})(4-z)}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(\lambda+1)}\right]^{\frac{(3-z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(\tilde{\zeta}+1)(4-z)(1+\lambda)}}, \quad (27)$$

where δ_2 and $G[N]$ are defined by $F_2 = F_1\varpi^{\frac{z-1}{4-z}}$ and $G[N] = [\exp(\frac{3N}{g(\tilde{m}+2)} + (\frac{3}{\tilde{m}+2})fg)^{\frac{g}{g-1}}]^{\frac{1}{g}}$. The scalar spectral index n_s is defined as $n_s = 1 + \frac{d\ln P_R}{d\ln\kappa}$, and by combining Eqn. (3.1) and (3.11), we get

$$n_s = 1 + \frac{(3z-11)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(3-z)}{fg(1+\tilde{\zeta})(4-z)(ln[t])^{g-1}} + \frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)+(z-3)(1+\tilde{\zeta})](g-1)}{fg(1+\tilde{\zeta})(4-z)(ln[t])^{g}} + n_2 + n_3, \quad (28)$$

Where n_2 and n_3 are terms of above equation can be obtained by

$$n_2 = \frac{1-z}{4-z}\sqrt{\frac{2(1+2\tilde{m})}{fg\kappa(\tilde{m}+2)(1+\tilde{\zeta})}}\left(\frac{(1+2\tilde{m})(fg)^2}{\kappa}\right)^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \times (ln[t])^{\frac{(1-g)(1+3\tilde{\zeta})}{2}} t^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}} \times \left[1 - \frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})f^2g^2(ln[t])^{2(g-1)}}{\kappa t^2}\right)^{-(\lambda+1)}\right]^{\frac{-(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{2(\tilde{\zeta}+1)(1+\lambda)}}$$

and

$$n_3 = \left(\frac{-2\tilde{\chi}}{(1+\tilde{\zeta})^2}\right)\left(\frac{3-z}{4-z}\right)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))\frac{(\kappa/(2\tilde{m}+1))^{1+\lambda}}{(\tilde{\alpha}g)^{3+2\lambda}}$$
$$\times\ (\frac{1}{ln[t]})^{(3+2\lambda)(g-1)}(t)^{2(1+\lambda)}\left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(t)^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2}\right)^{1+\lambda}\right]^{-1}$$

The scalar spectral index in the form of N

$$n_s = 1+\frac{(3z-11)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(3-z)}{fg(1+\tilde{\zeta})(4-z)(ln(G[N]))^{g-1}}$$
$$+\ \frac{(g-1)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(3-z)+(z-3)(1+\tilde{\zeta})](g-1)}{fg(1+\tilde{\zeta})(4-z)(ln(G[N]))^{g}}$$
$$+\ n_2+n_3, \tag{29}$$

Where n_2 and n_3 are defined as

$$n_2 = \frac{1-z}{4-z}\sqrt{\frac{2(1+2\tilde{m})}{fg\kappa(\tilde{m}+2)(1+\tilde{\zeta})}}\left(\frac{(1+2\tilde{m})(fg)^2}{\kappa}\right)^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}}$$
$$\times\ (ln(G[N]))^{\frac{(1-g)(1+3\tilde{\zeta})}{2}}(G(N))^{\frac{-\tilde{\zeta}}{1+\tilde{\zeta}}}$$
$$\times\ \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{(1+2\tilde{m})f^2g^2(ln(G[N]))^{2(g-1)}}{\kappa(G[N])^2}\right)^{-(\lambda+1)}\right]^{\frac{-(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{2(\tilde{\zeta}+1)(1+\lambda)}}$$

and

$$n_3 = \left(\frac{-2\tilde{\chi}}{(1+\tilde{\zeta})^2}\right)\left(\frac{3-z}{4-z}\right)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))\frac{(\kappa/(2\tilde{m}+1))^{1+\lambda}}{(fg)^{3+2\lambda}}$$
$$\times\ (\frac{1}{(ln(G[N])})^{(3+2\lambda)(g-1)}(G[N])^{2(1+\lambda)}$$
$$\times\ \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(ln(G[N]))^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2}\right)^{1+\lambda}\right]^{-1}$$

The tensor perturbations is written as in form of standard scalar inflation Ref.([44]), we may compute the tensor to scalar ratio is $r=\frac{P_g}{P_R}$, we obtained,

$$P_g = 8\kappa\left(\frac{H_2}{2\pi}\right)^2(\frac{\tilde{m}+2}{3}), \tag{30}$$

$$r(\varphi) = \frac{2(\tilde{m}+2)\kappa f^2g^2}{3\pi^2F_1}\varphi^{\frac{z-1}{4-z}}t^{\frac{(11-3z)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(z-3)+2(1+\tilde{\zeta})(4-z)}{(1+\tilde{\zeta})(4-z)}}$$
$$\times\ (ln[t])^{\frac{(1-g)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+(3-z)(1+\tilde{\zeta})+2(4-z)(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(4-z)}}$$
$$\times\ \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa t^2(ln[t])^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2}\right)^{1+\lambda}\right]^{\frac{(z-3)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4-z)(1+\lambda)}} \tag{31}$$

Tensor to scalar ratio in terms of number of e-folds N ,

$$\begin{aligned} r &= \frac{2(\tilde{m}+2)\kappa f^2 g^2}{3\pi^2 F_1}(\tilde{\tau}(G[N]))^{\frac{z-1}{4-z}}(G[N])^{\frac{(11-3z)(1+\tilde{\zeta})+2(1+\tilde{\zeta})(3-z)+2\tilde{\zeta}(z-3)+2(1+\tilde{\zeta})(4-z)}{(1+\tilde{\zeta})(4-z)}} \\ &\times \ (ln(G[N]))^{\frac{(1-g)[(11-3z)(1+\tilde{\zeta})+2\tilde{\zeta}(z-3)+(3-z)(1+\tilde{\zeta})+2(4-z)(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(4-z)}} \\ &\times \ \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(2\tilde{m}+1)\tilde{f}^2g^2}\right)^{1+\lambda}\right]^{\frac{(z-3)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4-z)(1+\lambda)}} \end{aligned} \tag{32}$$

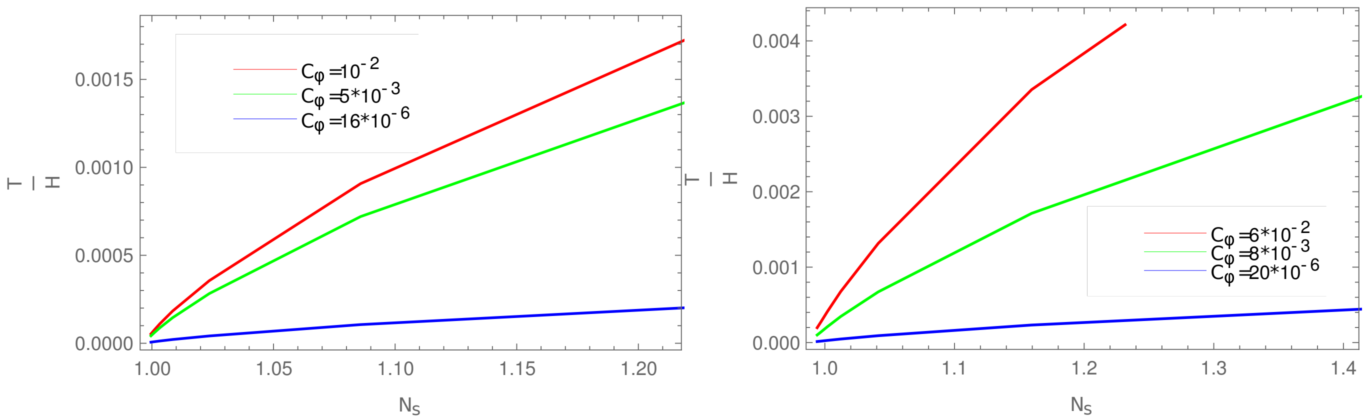

Figure 1:Plot of T/H verses n_s $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 15$, $f = 0.9805$, $\kappa = 1$

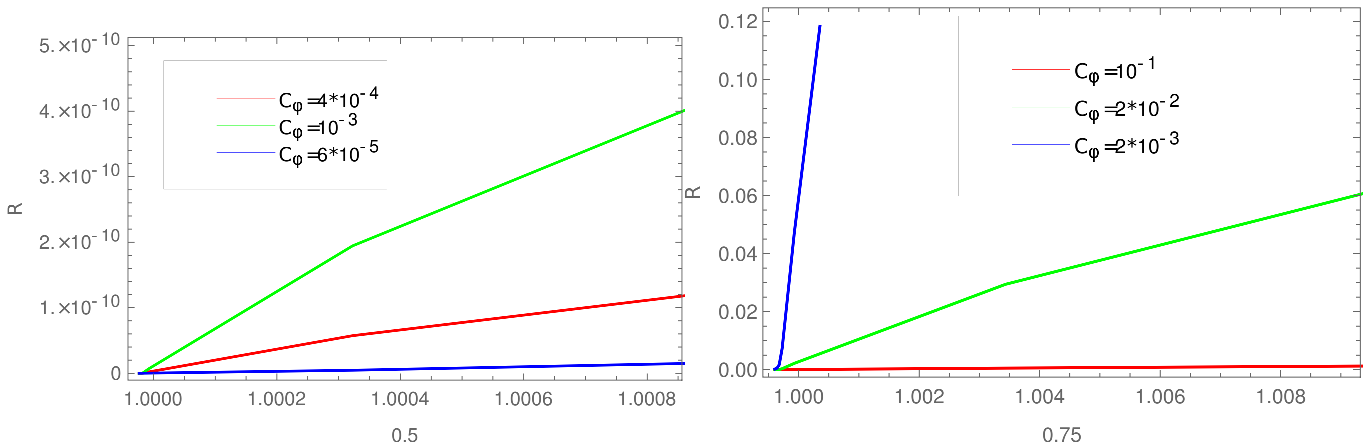

Figure 2; Plot of R verses n_s $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 15$, $f = 0.9805$, $\kappa = 1$

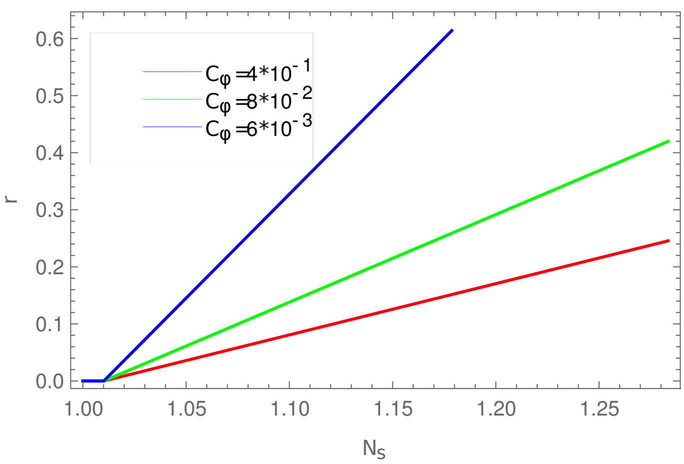

Figure 3: Plot of r versus n_s $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 15$, $f = 0.9805$, $\kappa = 1$

IV. THE STRONG DISSIPATIVE REGIME

In this section, we analyze the strong dissipative regime $\Gamma << (\tilde{m}+2)H_2$ in the scenario of inflationary model and discuss two special cases s.t $z = 3$ and $z \neq 3$. We formulated the solution of scalar filed as the function of cosmic time by combining Eq. (2.10) and (2.15),

$$\check{\varphi}(t) - \check{\varphi}_0 = \exp\left(\frac{\check{\tau[t]}}{\check{\varpi}}\right) \tag{33}$$

The term $\check{\varpi}$ and function $\check{\tau[t]}$ are defined as,

$$\check{\tau} = \frac{2^{\frac{1}{8}} \times 3^{\frac{3}{8}}}{(\tilde{m}+2)^{\frac{-3}{8}} \times C_{\varphi}^{\frac{-1}{2}} \times C_{\gamma}^{\frac{-3}{8}}}(\frac{1}{fg})^{\frac{5+3\tilde{\zeta}}{8(1+\tilde{\zeta})}}(\frac{\kappa}{1+2\tilde{m}})^{\frac{7+8\tilde{\zeta}}{8(1+\tilde{\zeta})}}$$

$$\begin{aligned}
\check{\tau[t]} &= (ln[t])^{\frac{(g-1)(\tilde{\zeta})}{8(1+\tilde{\zeta})}} \frac{-\tilde{\chi}t^{-2(1+\lambda)}(\tilde{\zeta}+\lambda+\tilde{\zeta}\lambda)}{(1+\lambda)(1+\tilde{\zeta})}(2)^{-\frac{(-1+f)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})}} \\
&\times Gamma[1 - \frac{(-1+g)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})} \\
&, \frac{(9+8\lambda+2\tilde{\zeta}(5+4\lambda))ln[t]}{4(1+\tilde{\zeta})}](ln[t])^{\frac{3+5\tilde{\zeta}+5g+3\tilde{\zeta}g}{8(1+\tilde{\zeta})}} \\
&\times \left(\frac{-(9+8\lambda+2\tilde{\zeta}(5+4\lambda))ln[t]}{(1+\tilde{\zeta})}\right)^{-\frac{(-1+g)(11+16\lambda+\tilde{\zeta}(13+16\lambda))}{4(1+\tilde{\zeta})}} \\
&\times \left(\frac{\kappa t^2(ln[t])^{2(1-g)}}{(fg)^2(1+2\tilde{m})}\right)^{1+\lambda} \\
&+ \left(\frac{8(1+\tilde{\zeta})^2}{(1+2\tilde{\zeta})}\right)(ln[t])^{\frac{(5+4\tilde{\zeta})(g-1)}{8(1+\tilde{\zeta})}}\left(-\frac{ln[t]+2\tilde{\zeta}ln[t]}{4(1+\tilde{\zeta})}\right)^{-\frac{(5+3\tilde{\zeta})(g-1)}{8(1+\tilde{\zeta})}} \\
&\times Gamma[\frac{(3+5\tilde{\zeta}+5\tilde{\zeta}+3\tilde{\zeta}g)}{8(1+\tilde{\zeta})}, -\frac{(1+2\tilde{\zeta})(ln[t])}{4(1+\tilde{\zeta})}]
\end{aligned}$$

The Hubble parameter as function of inflation field for $z = 3$ by using equation's (2.15) and (4.1)

$$H_2(\check{\varphi}) = \frac{fg}{(\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}])(ln(\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}]))^{1-g}} \tag{34}$$

In strong case, the scalar potential $V(\check{\varphi})$ is given by

$$V(\check{\varphi}) \approx \left[\left(\frac{(1+2\tilde{m})f^2g^2}{\kappa(\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}])^2(ln(\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}]))^{2(1-g)}}\right)^{1+\lambda} - \frac{\chi}{1+\zeta}\right]^{\frac{1}{(1+\zeta)(1+\lambda)}} \tag{35}$$

We analyze the constant dissipative coefficient for special case $z = 3$ by using Eqs. (2.1) and (4.1), we obtain the result as;

$$\begin{aligned}
\Gamma(\check{\psi}) &= F_3(\tilde{\tau}(G(N)))^{-2}(G(N))^{\frac{-3}{2(1+\tilde{\zeta})}}(ln(G(N)))^{\frac{3(g-1)(1-\tilde{\zeta})}{4(1+\zeta)}} \\
&\times \left[-\frac{\chi}{\zeta+1}\left(\frac{\kappa(G(N))^2(ln(G(N)))^{(-2g+2)}}{f^2(1+2\tilde{m})g^2}\right)^{1+\lambda}+1\right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)(1+\lambda)}}
\end{aligned} \tag{36}$$

Where $F_3 = C_{\check{\varphi}}\left[\frac{3(1+2\tilde{m})fg}{2\kappa C_\gamma(1+\zeta)(\tilde{m}+2)}\right]^{\frac{3}{4}}\left(\frac{(1+2\check{m})f^2g^2}{\kappa}\right)^{\frac{-3\zeta}{4(1+\zeta)}}$ is a constant term. By using eqn (2.1) and (4.1), the interaction between the quantity of e-folds N is found as,

$$\begin{aligned} N &= \left(\frac{\tilde{m}+2}{3}\right)\int_{t_1}^{t_2} H_2 dt \\ &= \frac{(\tilde{m}+2)f}{3}\left[(\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}_2])^g - (\check{\tau}^{-1}[\check{\varpi}\ln\check{\psi}_1])^g\right] \end{aligned} \tag{37}$$

According to the early universe, the thermal fluctuations provide the main source of scale density in scenario of inflationary. For the first time, the power spectrum is introduced by [45], when the friction coefficient in the inflation equation of motion depends on temperature. If the constant dissipative coefficient increases with temperature then the always increases the scalar perturbations. We discuss the high temperature case by ([46, 47]). For the strong dissipative regime $R \equiv \frac{\Gamma}{(\tilde{m}+2)H_2} > 1$. Where $< \delta\psi >$ is the scalar field fluctuation and can be written as $< \delta\psi >^2 \simeq \frac{\kappa_F T}{2\pi^2}$. The new function is known as wave number and it can defined κ_F this function is also will be equal to $\kappa_F = \sqrt{(\frac{\tilde{m}+2}{3})\Gamma H_2} = (\tilde{m}+2)\sqrt{\frac{H_2}{3}}$ after following at these Eqs. (2.1) ,(2.12) and (2.14), this equations will be equal to the scalar perturbation,

$$\begin{aligned} P_R &= (\frac{\tilde{m}+2}{3})^{\frac{5}{2}}\frac{H^{\frac{5}{2}}\Gamma^{\frac{1}{2}}T}{2\pi^2\dot{\check{\psi}}^2} = (\frac{\tilde{m}+2}{3})^{\frac{5}{2}}\frac{\kappa(1+\zeta)C_{\check{\psi}}^{\frac{3}{2}}\phi^{-3}}{4\pi^2(1+2\tilde{m})}\left(\frac{1+2\tilde{m}}{\kappa}\right)^{\frac{-3\zeta}{8(1+\zeta)}} \\ &\times\ H_2^{\frac{3(2+\zeta)}{4(1+\zeta)}}(-\dot{H}_2)^{\frac{3}{8}}\left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)}\right]^{\frac{11}{8}} \\ &\times\ \left[1-\frac{\chi}{1+\zeta}\left(\frac{(1+2\tilde{m})H_2^2}{\kappa}\right)^{-(1+\lambda)}\right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{8(1+\zeta)(1+\lambda)}} \end{aligned} \tag{38}$$

Another form of scalar perturbation can be written as in scalar $\check{\phi}$ perturbation $\check{\varphi}$;

$$\begin{aligned} P_R &= F_4(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^{-\frac{3(3+2\tilde{\zeta})}{4(1+\tilde{\zeta})}}(ln(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}]))^{\frac{3(g-1)(5+3\zeta)}{8(1+\zeta)}}\check{\varphi}^{-3} \\ &\times\ \left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^2(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{2(2+2\zeta)(2+2\lambda)}} \end{aligned} \tag{39}$$

where

$$\begin{aligned} F_4 &= \left(\frac{\tilde{m}+2}{3}\right)^{\frac{5}{2}}\frac{\kappa C_{\check{\psi}}^{\frac{3}{2}}(1+\zeta)}{(1+2\tilde{m})4\pi^2}\left(\frac{1+2\tilde{m}}{\kappa}\right)^{\frac{-3\zeta}{8(1+\zeta)}} \\ &\times\ (fg)^{\frac{2[3(5+3\zeta)}{8(1+\zeta)}}\left[\frac{3(1+2\tilde{m})}{2(\tilde{m}+2)\kappa C_\gamma(1+\zeta)}\right]^{\frac{11}{8}} \end{aligned}$$

The power spectrum written as a number of e-folds for $z = 3$

$$\begin{aligned} P_R &= F_4(G[N])^{-\frac{3(3+2\tilde{\zeta})}{4(1+\tilde{\zeta})}}(ln(G[N]))^{\frac{3(g-1)(5+3\zeta)}{8(1+\zeta)}}\exp\left(\frac{-3}{\check{\omega}}(\check{\tau}(G[N]))\right) \\ &\times\ \left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa(G[N])^2(\check{G[N]})^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{-3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)(2+2\lambda)}}, \end{aligned} \tag{40}$$

Considering the Eqs. (4.7) and (4.5) ;

$$n_s = 1 - \frac{3(3+2\tilde{\zeta})}{4fg(1+\tilde{\zeta})(ln[t])^{g-1}} + \frac{3(g-1)(5+3\zeta)}{8fg(1+\zeta)(ln[t])^g} + n_1 + n_2 \tag{41}$$

where n_1 and n_2 are the next two terms of spectral index

$$\begin{aligned} n_1 &= -3\left(\frac{2(1+2\tilde{m})}{\kappa(1+\zeta)}\right)^{\frac{1}{2}}\left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)}\right]^{\frac{-3}{8}}\frac{(fg)^{\frac{-3-4\tilde{\zeta}}{8(1+\tilde{\zeta})}}}{C_\varphi^{\frac{1}{2}}} \\ &\times \left(\frac{(2\tilde{m}+1)}{\kappa}\right)^{\frac{-\zeta}{8(1+\zeta)}}\left(ln(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])\right)^{\frac{(1-g)(3+4\tilde{\zeta})}{8(1+\tilde{\zeta})}}\left((\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}]\right)^{\frac{1+2\tilde{\zeta}}{4(1+\tilde{\zeta})}} \\ &\times \left[1-\frac{\chi}{1+\zeta}\left(\frac{(ln(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}]))^{2(1-g)}((\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^2\kappa}{(1+2\tilde{m})f^2g^2}\right)\right]^{\frac{-(\zeta+\lambda(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \end{aligned} \quad (42)$$

and

$$\begin{aligned} n_2 &= \frac{3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)}\left(\frac{\tilde{\chi}}{1+\tilde{\zeta}}\right)\left(\frac{(\frac{\kappa}{1+2\tilde{m}})^{1+\lambda}}{(fg)^{3+2\lambda}}\right)(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^{2(1+\lambda)} \\ &\times (ln(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}]))^{(3+2\lambda)(1-g)} \\ &\times \left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa((\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^2(\check{\tau}^{-1}[\check{\omega}\ln\check{\varphi}])^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)\right]^{-1} \end{aligned} \quad (43)$$

Similarly, can be written as number of e-folds of spectral index as given ,

$$\begin{aligned} n_s &= 1-\frac{3(3+2\tilde{\zeta})}{4fg(1+\tilde{\zeta})(ln(G[N])^{g-1}}+\frac{3(g-1)(5+3\zeta)}{8fg(1+\zeta)(ln(G[N]))^g} \\ &+ n_1+n_2 \end{aligned} \quad (44)$$

where

$$\begin{aligned} n_1 &= -3\left(\frac{2(1+2\tilde{m})}{\kappa(1+\zeta)}\right)^{\frac{1}{2}}\left[\frac{3(1+2\tilde{m})}{2\kappa C_\gamma(\tilde{m}+2)(1+\zeta)}\right]^{\frac{-3}{8}}\frac{(fg)^{\frac{-3-4\tilde{\zeta}}{8(1+\tilde{\zeta})}}}{C_\varphi^{\frac{1}{2}}} \\ &\times \left(\frac{(2\tilde{m}+1)}{\kappa}\right)^{\frac{-\zeta}{8(1+\zeta)}}(ln(G[N]))^{\frac{(1-g)(3+4\tilde{\zeta})}{8(1+\tilde{\zeta})}}(G[N])^{\frac{1+2\tilde{\zeta}}{4(1+\tilde{\zeta})}} \\ &\times \left[1-\frac{\chi}{1+\zeta}\left(\frac{(ln(G[N]))^{2(1-g)}(G[N])^2\kappa}{(1+2\tilde{m})f^2g^2}\right)\right]^{\frac{-(\zeta+\lambda(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \end{aligned} \quad (45)$$

and

$$\begin{aligned} n_2 &= \frac{3(\zeta+\lambda(1+\zeta))}{4(1+\zeta)}(\frac{\tilde{\chi}}{1+\tilde{\zeta}})\left(\frac{(\frac{\kappa}{1+2\tilde{m}})^{1+\lambda}}{(fg)^{3+2\lambda}}\right)(G[N])^{2(1+\lambda)} \\ &\times (ln(G[N]))^{(3+2\lambda)(1-g)} \\ &\times \left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)\right]^{-1} \end{aligned} \quad (46)$$

The tensor perturbations can be written as $\check{\varphi}$

$$\begin{aligned} r &= (\frac{\tilde{m}+2}{3})\frac{2\kappa(fg)^2}{\pi^2F_4}(t)^{\frac{(1-2\tilde{\zeta})}{4(1+\tilde{\zeta})}}(ln[t])^{\frac{(g-1)(1+7\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\ &\times \check{\psi}^3\left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa t^2(ln[t])^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{3(\tau+\lambda(1+\zeta)}{8(1+\zeta)(1+\lambda)}} \end{aligned} \quad (47)$$

As the weak regimen the tensor to scalar ratio in terms of number of e-folds becomes

$$
\begin{aligned}
r &= (\frac{\tilde{m}+2}{3})\frac{2\kappa(fg)^2}{\pi^2 F_4}(G[N])^{\frac{(1-2\tilde{\zeta})}{4(1+\tilde{\zeta})}}(ln(G[N]))^{\frac{(g-1)(1+7\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\
&\times \ \check{\psi}^3\left[1-\frac{\chi}{1+\zeta}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{3(\tilde{\zeta}+\lambda(1+\tilde{\zeta})}{8(1+\zeta)(1+\lambda)}} \qquad (48)
\end{aligned}
$$

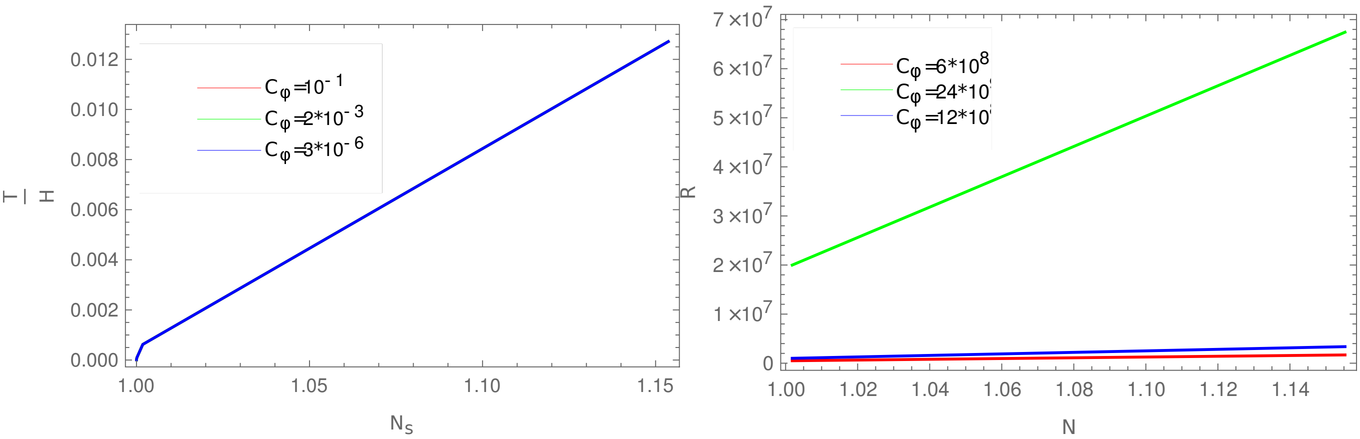

Figure 4: Plot T/H verses n_s (left) and R verses n_s (right) $\tilde{\zeta} = 0.0046$, $\tilde{\chi} = 0.8289$, $\tilde{m} = 1$, $g = 20$, $f = 0.9805$, $\kappa = 1$

V. THE STRONG DISSIPATIVE REGIME (SPECIAL CASE $z \neq 3$)

In this case, we discuss the scalar field for a special case of a strong regimen for $z \neq 3$, we obtained

$$\hat{\varphi}(t) - \hat{\varphi}_0 = \frac{\hat{\tau}_n[t]}{\hat{\omega}_n} \qquad (49)$$

where $\hat{\phi}$ is another new scalar field which is defined as $\phi[t] = \frac{2}{3-z}\hat{\varphi}^{\frac{3-z}{2}}$ and also $\hat{\omega}_n$ and $\hat{\tau}_n[t]$ both function written as,

$$\hat{\omega}_n = \frac{3^{\frac{z}{8}}}{2^{\frac{4+z}{8}}}\frac{C_{\hat{\varphi}}^{\frac{1}{2}}}{C_\gamma^{\frac{n}{8}}}\left(\frac{1}{1+\tilde{\zeta}}\right)^{\frac{z-4}{8}}(\frac{1}{\tilde{m}+2})^{\frac{z}{8}}(\frac{1}{fg})^{\frac{8-z+\tilde{\zeta}z}{8(1+\tilde{\zeta})}}\left(\frac{\kappa}{1+2\tilde{m}}\right)^{\frac{4-z}{8(1+\tilde{\zeta})}}$$

and

$$\begin{aligned}\hat{\tau}_n[t] &= \left(\frac{4(1+\tilde{\zeta})}{(2(\tilde{\zeta}-1)+z)}\right)\left(\frac{2(1-\tilde{\zeta})+z}{4(1+\tilde{\zeta})}\right)^{-\frac{(g-1)(8+z(\tilde{\zeta}-1))}{8(1+\tilde{\zeta})}}\\
&\times\ Gamma\left[1+\frac{(g-1)(8+z(\tilde{\zeta}-1))}{8(1+\tilde{\zeta})},-\frac{z+2(\tilde{\zeta}-1)ln[t]}{4(1+\tilde{\zeta})}\right]\\
&-\ \left(\frac{\tilde{\chi}\kappa(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+2\tilde{m})(fg)^2(1+\lambda)(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))^3}\right)\\
&\times\ (2)^{3-\frac{8+(-1+\tilde{\zeta})z-16(1+\tilde{\zeta})\lambda+g(8+z-\tilde{\zeta}z+16\lambda+16\tilde{\zeta}(1+\lambda))}{8(1+\tilde{\zeta})}}\\
&\times\ Gamma[\frac{(16(1+\lambda)+z)-g(8+z+16\lambda)}{4(1+\tilde{\zeta})}\\
&+\ \frac{\tilde{\zeta}(24-z+16\lambda+f(z-16(1+\lambda)))}{8(1+\tilde{\zeta})},\frac{(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))ln[t]}{4(1+\tilde{\zeta})}]\\
&\times\ (ln[t])^{-\frac{8+(-1+\tilde{\zeta})z+g(8-\tilde{\zeta}(-16+z)+z)}{8(1+\tilde{\zeta})}}\\
&\times\ \left(-\frac{(6+z+8\lambda+2\tilde{\zeta}(5+4\lambda))ln[t]}{1+\tilde{\zeta}}\right)^{\frac{8+(-1+\tilde{\zeta})z-16(1+\tilde{\zeta})\lambda+g(8+z-\tilde{\zeta}z+16\lambda+16\tilde{\zeta}(1+\lambda))}{8(1+\tilde{\zeta})}}\\
&\times\ \left(\frac{\kappa(ln[t])^{(2-2g)}}{(1+2\tilde{m})(fg)^2}\right)^{\lambda}\end{aligned}$$

According to this case, The definition of Hubble parameter is define,

$$H_2(\hat{\varphi})=\frac{fg}{((\hat{\tau}^{-1}[\hat{\omega}\hat{\varphi}]))(ln(\hat{\tau}^{-1}[\hat{\omega}\hat{\varphi}]))^{1-g}} \tag{50}$$

For special second case $z\neq 3$, The Potential in this form

$$V(\hat{\varphi})\approx\left[\left(\frac{(1+2\tilde{m})f^2g^2}{\kappa(G[N])^2(ln(G[N])^{2(1-g)}}\right)^{1+\lambda}-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\right]^{\frac{1}{(1+\tilde{\zeta})(1+\lambda)}} \tag{51}$$

For this case, the dissipative coefficient after the solved can be written as,

$$\begin{aligned}\Gamma(\hat{\varphi}) &= F_n\hat{\varphi}^{-2}(\tau_n^{-1}[\varpi_n\ln\hat{\varphi}])^{\frac{2z\tilde{\zeta}(1-g)-z(2-g)(1+\tilde{\zeta})}{4(1+\tilde{\zeta})}}\\
&\times\left[-\frac{\tilde{\chi}}{\tilde{\zeta}+1}\left(\frac{\kappa(\tau_n^{-1}[\varpi_n\ln\hat{\varphi}])^{2(1-g)}}{f^2(2\tilde{m}+1)g^2}\right)^{1+\lambda}+1\right]^{\frac{-z(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{(1+\tilde{\zeta})(4\lambda+4)}}\end{aligned} \tag{52}$$

and the constant term, where $F_z=C_{\hat{\varphi}}\left[\frac{3(1+2\tilde{m})\tilde{\alpha}g(1-g)}{2(\tilde{m}+2)\kappa C_\gamma(1+\tilde{\zeta})}\right]^{\frac{z}{4}}\left(\frac{(1+2\tilde{m})f^2g^2}{\kappa}\right)^{\frac{-z\tilde{\zeta}}{4(1+\tilde{\zeta})}}$.

For this case, The number of e-folds ,we get

$$N=f\left(\frac{m+2}{3}\right)\left[(\tau_n^{-1}[\varpi_n\ln\hat{\varphi}_2])^g-(\tau_n^{-1}[\varpi_n\ln\hat{\varphi}_1])^g\right]. \tag{53}$$

In special second case for $z\neq 3$, The Power Spectrum can be found that

$$\begin{aligned}P_R &= F_{1n}\hat{\varphi}^{\frac{3(1-z)}{2}}(ln[t])^{\frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}}(\frac{1}{t})^{\frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}}\\
&\times\left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa t^2(ln(t))^{2(1-g)}}{(2\tilde{m}+1)f^2g^2}\right)^{1+\lambda}\right]^{\frac{(6-3z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(4-z)(1+\lambda)}}\end{aligned} \tag{54}$$

Where,

$$F_{1n} = \left[\frac{(\tilde{m}+2)}{3}\right]^{\frac{5}{2}} \frac{\kappa(1+\tilde{\zeta})}{2(1+2\tilde{m})} C_{\hat{\varphi}}^{\frac{3}{2}} \left(\frac{1+2\tilde{m}}{\kappa}\right)^{\frac{(6-3z)\tilde{\zeta}}{8(1+\tilde{\zeta})}} \left[\frac{3(1+2\tilde{m})}{2\kappa C_{\gamma}(\tilde{m}+2)(1+\tilde{\zeta})}\right]^{\frac{3z+2}{8}} \times (fg)^{\frac{[(1+3\tilde{\zeta})(6-3z)-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}}. \quad (55)$$

Consequently, The power spectrum is defined number of e-folds, we get

$$P_R = F_{1n}(\tilde{\tau}_n(G[N]))^{\frac{3(1-z)}{2}} (ln(G[N]))^{\frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \left(\frac{1}{G[N]}\right)^{\frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \times \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(2\tilde{m}+1)f^2g^2}\right)^{1+\lambda}\right]^{\frac{(6-3z)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(4-z)(1+\lambda)}} \quad (56)$$

Where γ_n is a constant term and is defined by $\gamma_n = (\frac{1}{\delta_n})^{\frac{3-3z}{2}}$ and also written as of scalar spectrum index in scalar field

$$n_s = 1 + \frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8fg(1+\tilde{\zeta})(ln[t])^g} - \frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8fg(1+\tilde{\zeta})(ln[t])^{g-1}} + n_{1n} + n_{2n}, \quad (57)$$

where, The terms n_{1n} and n_{2n} are

$$n_{1n} = \left(\frac{3(1-z)}{2C_{\hat{\varphi}}^{\frac{1}{2}}}\right)\left(\frac{2(1+2\tilde{m})}{\kappa(1+\tilde{\zeta})}\right)^{\frac{1}{2}}\left[\frac{3(1+2\tilde{m})fg}{2C_{\gamma}\kappa(\tilde{m}+2)(1+\tilde{\zeta})}\right]^{\frac{-z}{8}} \times \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa}\right)^{\frac{\tilde{\zeta}(z-4)}{8(1+\tilde{\zeta})}} \hat{\varphi}^{\frac{z-3}{2}} (ln[t])^{\frac{(g-1)[2\tilde{\zeta}(z-4)-\tilde{m}(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \times (t)^{\frac{(\tilde{m}-4)(1-\tilde{\zeta})}{8(1+\tilde{\zeta})}}\left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa t^2(ln[t])^{2(1-g)}}{(1+2\tilde{m})\tilde{\alpha}^2g^2}\right)^{1+\lambda}\right]^{\frac{(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}}$$

and

$$n_{2n} = -\frac{(6-3z)\tilde{\chi}}{4(1+\tilde{\zeta})}\frac{[\tilde{\zeta}+\lambda(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(fg)^{3+2\lambda}} \times (\kappa/(1+2\tilde{m}))^{1+\lambda}(t)^{2(1+\lambda)}\left(\frac{1}{ln[t]}\right)^{(g-1)(3+2\lambda)} \times \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(t)^2(ln[t])^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{-1}.$$

In this case $z \neq 3$ and the scalar spectral index which can be expressed in number of e-folds,

$$n_s = 1 + \frac{(g-1)[(6-3z)(1+3\tilde{\zeta})-8(1+\tilde{\zeta})]}{8fg(1+\tilde{\zeta})(ln(G[N]))^g} - \frac{2(6-3z)(1+2\tilde{\zeta})-8(1+\tilde{\zeta})}{8fg(1+\tilde{\zeta})(ln(G[N]))^{g-1}} + n_{1n} + n_{2n}, \quad (58)$$

where n_{1n} and n_{2n} are

$$
\begin{aligned}
n_{1n} &= \left(\frac{3(1-z)}{2C_{\varphi}^{\frac{1}{2}}}\right)\left(\frac{2(1+2\tilde{m})}{\kappa(1+\tilde{\zeta})}\right)^{\frac{1}{2}}\left[\frac{3(1+2\tilde{m})fg}{2C_{\gamma}\kappa(\tilde{m}+2)(1+\tilde{\zeta})}\right]^{\frac{-z}{8}} \\
&\times \left(\frac{(1+2\tilde{m})f^2g^2}{\kappa}\right)^{\frac{\tilde{\zeta}(z-4)}{8(1+\tilde{\zeta})}}\hat{\varphi}^{\frac{z-3}{2}}(ln(G[N]))^{\frac{(g-1)[2\tilde{\zeta}(z-4)-\tilde{m}(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}} \\
&\times (G[N])^{\frac{(\tilde{m}-4)(1-\tilde{\zeta})}{8(1+A)}}\left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})\tilde{\alpha}^2g^2}\right)^{1+\lambda}\right]^{\frac{(z-4)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}}
\end{aligned}
$$

and

$$
\begin{aligned}
n_{2n} &= -\frac{(6-3z)\tilde{\chi}}{4(1+\tilde{\zeta})}\frac{[\tilde{\zeta}+\lambda(1+\tilde{\zeta})]}{(1+\tilde{\zeta})(fg)^{3+2\lambda}} \\
&\times (\kappa/(1+2\tilde{m}))^{1+\lambda}(G[N])^{2(1+\lambda)}\left(\frac{1}{ln(G[N])}\right)^{(g-1)(3+2\lambda)} \\
&\times \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{2(1-g)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{-1}.
\end{aligned}
$$

In the second case, similarly we also find the tensor-to-scalar ratio,

$$
\begin{aligned}
r &= \frac{2\kappa(fg)^2}{\pi^2F_n}\left(\frac{\tilde{m}+2}{3}\right)\hat{\varphi}^{\frac{3(z-1)}{2}} \\
&\times (ln[t])^{\frac{(1-g)[(6-3z)(3\tilde{\zeta}+1)-16(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}}(t)^{\frac{2(1+2\tilde{\zeta})(6-3z)-16(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\
&\times \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(t)^2(ln[t])^{-2(g-1)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{(3z-6)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}},
\end{aligned}
\tag{59}
$$

Similarly, this equation can also be written as number of e-folds

$$
\begin{aligned}
r &= \frac{2\kappa(fg)^2}{\pi^2F_n}\left(\frac{\tilde{m}+2}{3}\right)\hat{\varphi}^{\frac{3(z-1)}{2}} \\
&\times (ln(G[N]))^{\frac{(1-g)[(6-3z)(3\tilde{\zeta}+1)-16(1+\tilde{\zeta})]}{8(1+\tilde{\zeta})}}(G[N])^{\frac{2(1+2\tilde{\zeta})(6-3z)-16(1+\tilde{\zeta})}{8(1+\tilde{\zeta})}} \\
&\times \left[1-\frac{\tilde{\chi}}{1+\tilde{\zeta}}\left(\frac{\kappa(G[N])^2(ln(G[N]))^{-2(g-1)}}{(1+2\tilde{m})f^2g^2}\right)^{1+\lambda}\right]^{\frac{(3z-6)(\tilde{\zeta}+\lambda(1+\tilde{\zeta}))}{8(1+\tilde{\zeta})(1+\lambda)}}.
\end{aligned}
\tag{60}
$$

VI. CONCLUSION

In this present work, we have studied the warm anisotropic inflationary universe with modified Chaplygin gas in the background of locally rotationally symmetric Bianchi I universe model. We evaluate the inflationary expansion of universe via constant dissipative coefficient $\Gamma = C_{\varphi}T^z/\varphi^{z-1}$, where $z = -1, 0, 1$ and 3. We study the possible relaization of an expanding logamediate scale factor in weak and strong dissipative regime. Under the slow roll approximation, we evaluate the scalar power spectrum, scalar power index and tensor to scalar ratio. For both regimes, we have found the constraints on several parameters, considering the Planck 2018 data, together with the condition for warm inflation $T > H_2$ and the condition for the weak $\Gamma < (m+2)H_2$ (or strong $\Gamma > (m+2)H_2$) dissipative regime. For the weak dissipative regime, we have obtained the constraints on the parameters of our model, only from the conditions $T > H_2$, which gives an upper bound, and $T < H_2$, which gives a lower bound. This is due that fact that the consistency relation $r = r(n_s)$ does not impose constraints on the parameters by considering two-marginalized constraints at 68% and 95% C.L. For the strong dissipative regime, we have found the constraints on the parameters from the Planck 2018 data, through the consistency relation $r = r(n_s)$, with condition of inflation $T > H_2$. According to recent observational Planck data, the strong dissipative regime for special case $z = 3$, the conditions of warm inflation $T > H_2$ model evolves under the regime and for two-dimensional marginalized constraints on the parameter r and n_s. Moreover, the constant dissipative parameter C_φ by the set of upper and lower limit. Finally, the values of $z = -1$ and $z = 0$ does not satisfy the condition of warm inflation in the case of strong dissipative regime and the plot of r verses n_s can not be draw in a strong case, since the predicted value of spectral index is always greater than unity. It is interesting that the recent observational data is also compatible with our inflationary dynamic model for specific value of tensor to scalar ratio $r \sim 0$. Finally, it is concluded that warm anisotropic inflationary scenario can be successfully discussed in the context of MCG and BI models with particular forms of generalized dissipation coefficient. In weak dissipative regime, we have obtained consistent results when dissipation coefficient corresponds to super-symmetric models ($z = 1$) while this consistency is preserved in strong dissipative regimes for $z = 3$. In case of non-supersymmetric models ($z = 1$), the warm inflationary condition is violated in both weak and strong dissipative regimes.

Acknowledgments

This work is supported by Comisión Nacional de Ciencias y Tecnología of Chile (CONICYT) through FONDECYT Grant No 3170035 (A. Ö.).

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