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Weak gravitational lensing by Bocharova–Bronnikov–Melnikov–Bekenstein black holes using Gauss–Bonnet theorem

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Abstract In this article, we demonstrate the weak gravitational lensing in the context of Bocharova–Bronnikov–Melnikov–Bekenstein (BBMB) black hole. To this desire, we derive the deflection angle of light in a plasma medium by BBMB black hole using the Gibbons and Werner method. First, we obtain the Gaussian optical curvature and implement the Gauss–Bonnet theorem to investigate the deflection angle for spherically symmetric spacetime of BBMB black hole. Moreover, we also analyze the graphical behavior of deflection angle by BBMB black hole in the presence of plasma medium.

1 Introduction

The discovery of first neutron collision [1] was an epochal which is the beginning of gravitational wave astronomy. After gravitational waves were discovered, a large number of the modified gravity theories face a lot of problems. On the other hand, one of the important testing methods of gravity is gravitational lensing. The light is deflected in the presence of a massive body which is predicted by general relativity. This phenomenon is called gravitational lensing. Its experimental conformation was first given by Cavendish, Soldner [2] as well as Einstein in their observation to identify the gravitational deflection of light. The gravitational lensing has been classified in the literature as a strong lensing, weak lensing and micro-lensing [3,4].

Weak gravitational lensing is a powerful tool to measure masses of a variety of objects in the universe. The weak gravitational lensing provides a way to find the mass of astronomical objects without requiring about their composition or dynamical states. Weak lensing also investigates the cause of the accelerated expansion of the universe and also distinguishes between modified gravity and dark energy. For instance, the lens equation is obtained for the Schwarzschild black hole by Ellis and Virbhadra [5] and more comprehensively discussed by Virbhadra [6]. Many authors investigate the gravitational lensing by other astrophysical

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objects such as naked singularities black holes (BH), wormholes and few other related objects [7–29].

Gibbons and Werner (GW) proposed a new method to calculate weak deflection angle using the Gauss–Bonnet theorem (GBT) [30]. They showed that this method provides that the deflection angle is a global property. Sakalli and Ovgun applied this novel method to the Rindler modified [31] Schwarzschild black hole and obtained the weak deflection angle. The Gibbons–Werner method has been applied by different authors on black holes as well as wormholes [32–58]. The GBT method is based on the following equation [30]:

$$\alpha = -\int\int_{D\infty} \mathcal{K} \mathrm{d}S.$$

Here, $\mathcal{K}$ and $\mathrm{d}S$ represent the Gaussian curvature and surface element, respectively. After that, Werner [59] extended this method and obtained the weak deflection of light by Kerr black hole by using the Nazims's method for Rander-Finsler metric. Recently, Gallo and Crisnejo (GC) [58] discussed the deflection of light in the plasma medium. Since then the work of weak gravitational lensing is continuous through the method of GW by using the GBT for different black holes and wormholes.

In this article, our main aim is to investigate weak deflection angle of BBMB black hole. For this purpose, this article is categorized as follows: In Sect. 2, we review the BBMB black hole. In Sect. 3, by using the GBT we compute the weak deflection angle of BBMB black hole in plasma medium. In Sect. 4, we conclude our results.

2 Bocharova–Bronnikov–Melnikov–Bekenstein black hole

The BBMB black hole in a static and spherically symmetric form is given as [60]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \tag{1}$$

where the metric function $f(r)$ is

$$f(r) = 1 - \frac{2m}{r} + \frac{m^2}{r^2}, \tag{2}$$

where m is a mass of BH. The optical space in equatorial plane ($\theta = \frac{\pi}{2}$) to get null geodesic ($ds = 0$)

$$\mathrm{d}t^2 = \frac{\mathrm{d}r^2}{f(r)^2} + \frac{r^2\mathrm{d}\phi^2}{f(r)}. \tag{3}$$

To investigate the weak gravitational lensing by BBMB black hole in plasma medium, we calculate the refractive index $n(r)$:

$$n(r) = \sqrt{1 - \frac{\omega_e^2}{\omega_\infty^2} f(r)}, \tag{4}$$

where ω_e and ω_∞ are the electron plasma frequency and light frequency calculated by an observer at infinity, respectively; then, the corresponding optical metric is illustrated as

$$\mathrm{d}\sigma^2 = g_{ij}^{opt}\mathrm{d}x^i\mathrm{d}x^j = \frac{n^2(r)}{f(r)}\left(\frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\phi^2\right). \tag{5}$$

The Gaussian optical curvature is evaluated as follows:

$$\mathcal{K} = \frac{\text{RicciScalar}}{2}. \tag{6}$$

After simplifying, Gaussian optical curvature for BBMB black hole in leading order term is calculated as follows:

$$\mathcal{K} \approx -\frac{2m}{r^3} - \frac{3m\omega_e^2}{\omega_\infty^2 r^3} + \frac{6m^2}{r^4} + \frac{17m^2\omega_e^2}{r^4\omega_\infty^2}. \tag{7}$$

3 Deflection angle of BBMB black holes and Gauss–Bonnet theorem

In this section, we drive the deflection angle of light by BBMB BH using the GBT. By using GBT in the region $\mathcal{H}_R$ with $\partial H_R = \gamma_{\tilde{g}} \cup C_R$, given as [30]

$$\int_{\mathcal{H}_R} \mathcal{K} \mathrm{d}S + \oint_{\partial\mathcal{H}_R} k \mathrm{d}t + \sum_t \epsilon_i = 2\pi\mathcal{X}(\mathcal{H}_R), \tag{8}$$

where k represents the geodesic curvature and $\mathcal{K}$ denotes the Gaussian optical curvature. One can define k as $\kappa = \tilde{g}\left(\nabla_{\dot{\gamma}}\dot{\gamma}, \ddot{\gamma}\right)$ in such a way that $\tilde{g}(\dot{\gamma}, \dot{\gamma}) = 1$, where $\ddot{\gamma}$ represents the unit acceleration vector and ϵ_i denotes the exterior angle at ith vertex, respectively. As $R \to \infty$, we obtain the jump angles equal to $\pi/2$; hence, total jump angles are $\epsilon_i + \epsilon_{ii} \to \pi$. Here, $\mathcal{X}(\mathcal{H}_R) = 1$ is a Euler characteristic number and $\mathcal{H}_R$ denotes the non-singular domain. Therefore, we obtain [30]

$$\int\int_{\mathcal{H}_R} \mathcal{K} \mathrm{d}S + \oint_{\partial\mathcal{H}_R} k \mathrm{d}t = \pi. \tag{9}$$

At 0^{th} order weak field deflection limit of the light for the straight-line approximation is defined as $r(t) = b/\sin\phi$. But we use the first-order light ray trajectory as follows [61]:

$$\frac{1}{r_p} = \frac{\sin(\phi)}{b} + \frac{1}{2}\frac{m(3+\cos(2\phi))}{b^2} + \frac{1}{16}\frac{m^2(37\sin(\phi) + 30(\pi - 2\phi)\cos(\phi) - 3\sin(3\phi))}{b} \tag{10}$$

and GW shows that the equation of GBT reduces to simple form [30]; then, GC analyzes the deflection angle in the plasma medium by using this equation [58]:

$$\alpha = -\int_0^\pi \int_{r_p}^\infty \mathcal{K}\sqrt{det g^{opt}} \mathrm{d}r d\phi; \tag{11}$$

here we put the leading term of equation Eq. 7 into above equation Eq. 11, so the obtained deflection angle of the photon rays is moving in a medium of homogeneous plasma up to leading order term computed as:

$$\alpha \simeq \frac{4m}{b} + \frac{2m\omega_e^2}{b\omega_\infty^2} - \frac{3m^2\pi}{4b^2}. \tag{12}$$

Hence, the effect of plasma can be removed if ($\omega_e = 0$), or ($\beta = \omega_e/\omega_\infty \to 0$), and it reduces to vacuum case, and it is in agreement with [62,63] up to the second order in m, if second term is Q^2 instead of m^2. The value of the photon frequency in plasma medium is $\omega_e/\omega_\infty = 6 \times 10^{-3}$ [64].

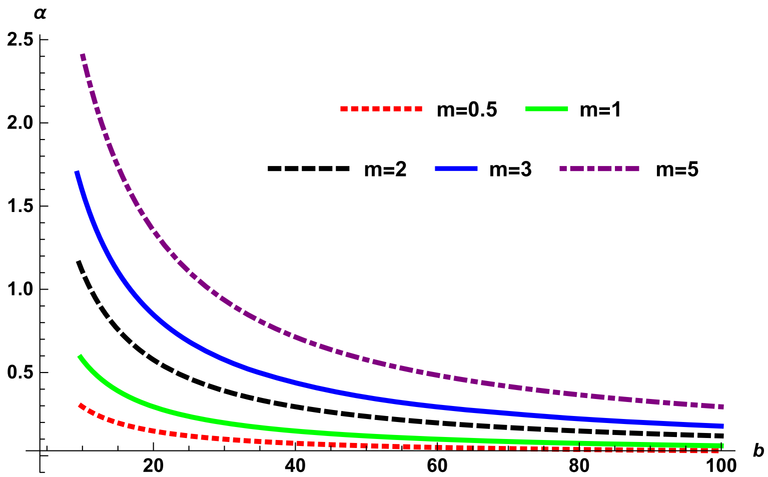

Fig. 1 α versus b by fixing the value of $\beta = 1$ and varying m

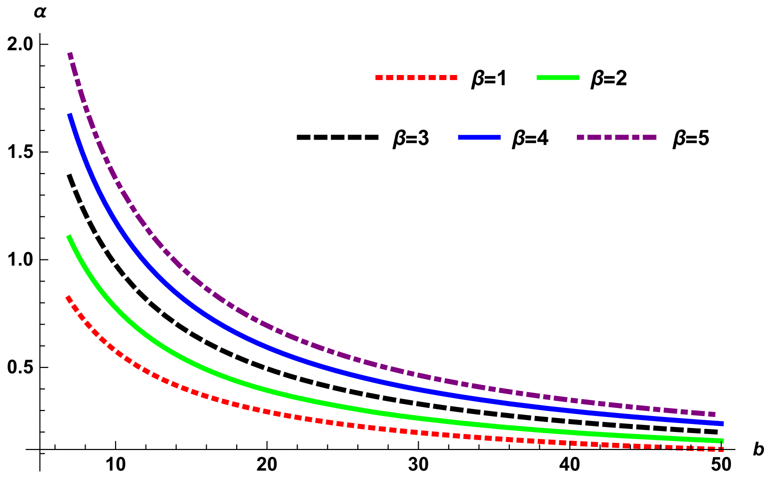

Fig. 2 α versus b by fixing the value of m and varying β

4 Conclusion

The present article is about the investigation of deflection angle by BBMB black hole in plasma medium. In this regard, we analyze the weak gravitational lensing by using GBT and get the deflection angle of light for BBMB black hole.

- Figure 1 indicates the behavior of deflection angle w.r.t b by fixing the value of β and varying m. It is to be observed that for values of increasing m, the behavior of deflection angle gradually increases.
- Figure 2 represents the behavior of deflection angle w.r.t b by varying the mass β and taking m fixed. We noticed that for values of $\beta > 0$ the behavior of deflection angle gradually increases, and on the other hand, for the values of $\beta < 0$ the behavior of deflection angle decreases.

The obtained deflection angle is given as follows:

$$\alpha \simeq \frac{4m}{b} + \frac{2m\omega_e^2}{b\omega_\infty^2} - \frac{3m^2\pi}{4b^2}. \tag{13}$$

We examine that by the reduction of some parameters the obtained deflection angle is converted into the Schwarzschild deflection angle up to the first-order terms. In addition we also discuss the graphical effect of different parameters on deflection angle by BBMB in a plasma medium.

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