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Weak gravitational lensing by stringy black holes

Wajiha Javed[1,a]**, Muhammad Bilal Khadim**[1,b]**, Jameela Abbas**[2,c]**, Ali Övgün**[3,4,d] (iD)

[1] Division of Science and Technology, University of Education, Township, Lahore, Pakistan
[2] Department of Mathematics, University of Education, Township, Lahore 54590, Pakistan
[3] Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4950, Valparaiso, Chile
[4] Physics Department, Arts and Sciences Faculty, Eastern Mediterranean University, Via Mersin 10, Famagusta, North Cyprus, Turkey

Abstract In this paper, we discuss the weak gravitational lensing in the context of stringy black holes. Initially, we examine the deflection angle of photon by charged stringy black hole. For this desire, we compute the Gaussian optical curvature and implement the Gauss–Bonnet theorem to investigate the deflection angle for spherically balanced spacetime of stringy black hole. We also analyze the influence of plasma medium in the weak gravitational lensing for stringy black hole. Moreover, the graphical impact of impact parameter b, black hole charge Q on deflection angle by charged stringy black hole has been studied in plasma as well as non-plasma medium.

1 Introduction

Einstein in 1916 announced about the presence of the gravitational lensing as well as gravitational waves as a part of the theory of general relativity (GR). His theory depends on many experimental and observational tests. In accordance with his theory, the force of gravity due to the fact of curvature in spacetime and gravitational waves [1] are ripples in the fabric of a universe which are obtained by the clashing of the gigantic objects such like black holes (BHs). The phenomenon in which gravity deflects light is referred as *gravitational lensing*. Gravitational lensing is very useful method to understand dark matter, galaxies and universe. After the first observation of Eddington, so many works have been done on gravitational lensing for black holes, wormholes and other objects. The topic of gravitational lensing takes a great attraction. Therefore, the gravitational lensing is categorized into two ways; strong gravitational lensing and other one is a weak gravitational lensing. The strong gravitational lensing tells us about the magnification, position and time delays of the images by BHs. Strong lensing phenomena also need in many cases which provide further information from experimental frame of reference to see the other different objects such as monopoles [2], boson stars [3–5], fermion stars [6,7], etc. On the other hand, the weak gravitational lensing gives a method to find the mass of astronomical objects without demanding to their forma-

[a] e-mails: wajiha.javed@ue.edu.pk; wajihajaved84@yahoo.com

[b] e-mail: blaljutt723@gmail.com

[c] e-mail: jameelaabbas30@gmail.com

[d] e-mail: ali.ovgun@pucv.cl (corresponding author)

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tion or dynamical states. Weak lensing also distinguishes between dark energy and modified gravity and also examines the cause of rapid expansion in the universe. The gravitational lensing is by means of the distribution of matter among the source and observer, that is qualified by deflection of light from the source to the observer also the deflection of light is one of the prediction of Einstein's theory [8–34]. The idea was first given by German astronomer Johan Georg Van Soldner 1801 in the background of Newtonian theory. In 1924 Chwolson and Einstein in 1936 derived the exact calculations of deflection of light in the framework of GR. Regarding to this, Gibbons and werner have investigated the deflection angle of light for asymptotically flat static BHs [35] by using Gauss–Bonnet theorem which is defined as:

$$\hat{\alpha} = -\int\int_{D_\infty} \mathcal{K} d\tilde{\sigma},$$

where $\mathcal{K}$ stand for Gaussian curvature and $d\tilde{\sigma}$ is a surface element of optical metric. After that, Werner expanded this strategy for stationary BHs [36]. Arakida and Kasai [37] have analyzed the engagement of cosmological constant on deflection of light also its status with in the cosmological lens equation. In recent times, Bisnovatyi-Kogan and Tsupko [38] have showed that the deflection angle in the homogeneous plasma is differed from the deflection angle in a vacuum, in addition that the deflection angle in plasma medium found on the photon frequency. Additionally, this procedure has been prolonged to the wormhole geometries and non-asymptotically flat spacetimes having topological defects [39–45].

According to GR, spacetime singularities rise various issues, both scientific and physical [46,47], by applying the nonlinear electrodynamics it is reasonable to solve these singularities by fabricating a regular BH solution [48–50]. He and Lin [51] have investigated the deflection of light for test particles, due to a axially moving Kerr–Newman BH with an arbitrary constant velocity that is perpendicular to its angular momentum. Additionally, it is demonstrated that here is no peculiarity of the electric field quality at the cause for the point-like particles and it has an attractive charge. By using the Gauss–Bonnet theorem GBT, deflection angle of light on the foundation of magnetized BH and impact of nonlinear electrodynamic has been found by Javed et al. [52,53]. Recently, plasma medium deflects photons shown by Crisnejo and Gallo [54]. By investigating the weak gravitational lensing for hairy BH in the back burner of Einstein–Maxwell theory (EMT) with a non-minimally coupled dilaton and its non-trivial potential, in this connection by virtue of GBT deflection angle of light has been computed with plasma and excluding plasma medium by Javed et al [55]. For more new works, one can see [55–84]. Main cause of this work is to study the stringy black holes by utilizing Gauss–Bonnet theorem and also to see the effect of topological defects on gravitational lensing. For examination, we consider the notation of the deflection angle of big particles and the deflection of photons in a plasma medium.

This paper arranged as: in Sect. 2, we concisely review regarding stringy black holes. In Sect. 3, we apply Gauss–Bonnet theorem to find deflection angle of stringy black holes. In Sect. 4, we enhance our work to find out the deflection angle in the presence of plasma medium and in addition we demonstrate the graphical impact of deflection angle in the context of stringy black hole. In Sect. 5, we recap our results which we obtain in present work.

2 Computation of weak lensing by stringy black holes and Gauss–Bonnet theorem

The stringy black holes metric in spherically coordinate is given as [85];

$$ds^2 = \frac{dt^2}{F(r)^2} - F(r)^2 \{dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}, \tag{1}$$

where $F(r)$ is

$$F(r) = 1 + \frac{Q}{r} + \frac{Q^2\alpha}{8r(r+Q)^3}. \tag{2}$$

Note that Q, and α is charge of stringy black hole and coupling constant, respectively. The optical metric is simply written in equatorial plane ($\theta = \frac{\pi}{2}$) to get null geodesics ($ds^2 = 0$)

$$dt^2 = F(r)^4 dr^2 + r^2 F(r)^4 d\phi^2. \tag{3}$$

The Gaussian optical curvature is evaluated as follows:

$$\mathcal{K} = \frac{\text{RicciScalar}}{2}. \tag{4}$$

After simplifying, Gaussian optical curvature is stated as:

$$\mathcal{K} \approx -2\frac{Q}{r^3} + \left(12\,r^{-4} - 4\frac{\alpha}{r^6}\right)Q^2 + \mathcal{O}(Q^3, \alpha^2, r^7). \tag{5}$$

Now, we derive the deflection angle of a stringy BH by using the GBT. By applying the GBT to the region $\mathcal{N}_R$, given as [35]

$$\int_{\mathcal{N}_R} \mathcal{K}dS + \oint_{\partial\mathcal{N}_R} kdt + \sum_k \sigma_k = 2\pi\mathcal{X}(\mathcal{N}_R), \tag{6}$$

where k is geodesic curvature, $\mathcal{K}$ represents Gaussian curvature, respectively, and k is defined as $k = \bar{g}(\nabla_{\tilde{\alpha}}\tilde{\alpha}, \bar{\alpha})$ in such a way that the Riemannian metric $\bar{g}(\tilde{\alpha}, \tilde{\alpha}) = 1$, here $\bar{\alpha}$ is unit acceleration vector and σ_k represents the exterior angle of kth vertex, respectively. When $R \to \infty$ the jump angles equal to $\pi/2$, hence $\sigma_O + \sigma_S \to \pi$. Where $\mathcal{X}(\mathcal{N}_R) = 1$ is a Euler characteristic and $\mathcal{N}_R$ represents the non-singular. Thus, we obtain

$$\int\int_{\mathcal{N}_R} \mathcal{K}dS + \oint_{\partial\mathcal{N}_R} kdt + \sigma_k = 2\pi\mathcal{X}(\mathcal{N}_R). \tag{7}$$

Here, $\alpha_{\tilde{g}}$ is a geodesic and the total jump angle is $\sigma_k = \pi$, since $\mathcal{X}$ represents Euler characteristic number which is 1. When $R \to \infty$, then remaining part is $k(D_R) = |\nabla_{\dot{D}_R}\dot{D}_R|$. Considering the radial component for geodesic curvature that is described as:

$$(\nabla_{\dot{D}_R}\dot{D}_R)^r = \dot{D}_R^\phi \partial_\phi \dot{D}_R^r + \Gamma_{\phi\phi}^r(\dot{D}_R^\phi)^2. \tag{8}$$

At very high R, $D_R := r(\phi) = R = const$. Thus, the leading term of above equation vanishes and $(\dot{D}_R^\phi)^2 = \frac{1}{f^2(r^\star)}$. Recalling $\Gamma_{\phi\phi}^r = \frac{-2r^2 F'}{F} - r$, we get

$$(\nabla_{\dot{D}_R^r}\dot{D}_R^r)^r \to \frac{1}{R}. \tag{9}$$

And which proves that the geodesic curvature is not effected to the topological defects (i.e., $k(D_R) \to R^{-1}$). We can write $dt = Rd\phi$. Thus,

$$k(D_R)dt = \frac{1}{R}Rd\phi. \tag{10}$$

From the previous results, we get

$$\int\int_{\mathcal{N}_R} \mathcal{K}\mathrm{d}s + \oint_{\partial\mathcal{N}_R} k\mathrm{d}t \overset{R\to\infty}{=} \int\int_{S_\infty} \mathcal{K}\mathrm{d}S + \int_0^{\pi+\sigma} \mathrm{d}\phi. \tag{11}$$

In the weak field deflection limit at the zeroth order, the light ray is described as $r(t) = b/\sin\phi$. Therefore, the deflection angle is stated as: [35]

$$\tilde{\alpha} = -\int_0^\pi \int_{b/\sin\phi}^\infty \mathcal{K}\sqrt{det\bar{g}}\mathrm{d}u\mathrm{d}\phi. \tag{12}$$

We substitute the leading term of Eq. 5 into above Eq. 12, so the obtained deflection angle up to leading order term is stated as:

$$\tilde{\alpha} = \frac{4Q}{b} - \frac{3Q^2\pi}{b^2} + \mathcal{O}(Q^3, b^3). \tag{13}$$

3 Graphical influence of deflection angle upon stringy black hole

In this portion, we analyze the graphical behavior of deflection angle $\tilde{\alpha}$ on stringy BH. We also give the physical importance of these graphs to examine the effect of impact parameter b, and black holes charge Q on deflection angle.

3.1 Deflection angle $\tilde{\alpha}$ w.r.t impact parameter b

- Figure 1, the left plot shows that the behavior of deflection angle σ w.r.t b for fixed BH charge Q has been checked.
  The right plot renders the behavior of deflection angle $\tilde{\alpha}$ with b by taking varying Q. We analyzed that the deflection angle exponentially decreased and approached to positive infinity. Thus, the deflection angle get the stable behavior for positive values.

4 Weak lensing by stringy black holes in a plasma medium

This segment is based on the investigation of weak gravitational lensing of stringy black hole in the presence of plasma medium. For stringy black holes, the refractive index $n(r)$ is obtained as, [73]

$$n(r) = \sqrt{1 - \frac{\omega_e^2}{\omega_\infty^2} F(r)^{-2}}, \tag{14}$$

where ω_e and ω_∞ are electron plasma frequency and photon frequency measured by an observer at infinity, respectively, then the corresponding optical metric is illustrated as

$$\mathrm{d}t^2 = g_{ij}^{\mathrm{opt}} dx^i dx^j = n^2(r)[F(r)^4 \mathrm{d}r^2 + r^2 F(r)^4 \mathrm{d}\phi^2]. \tag{15}$$

Here, the metric function of the above optical metric is given as

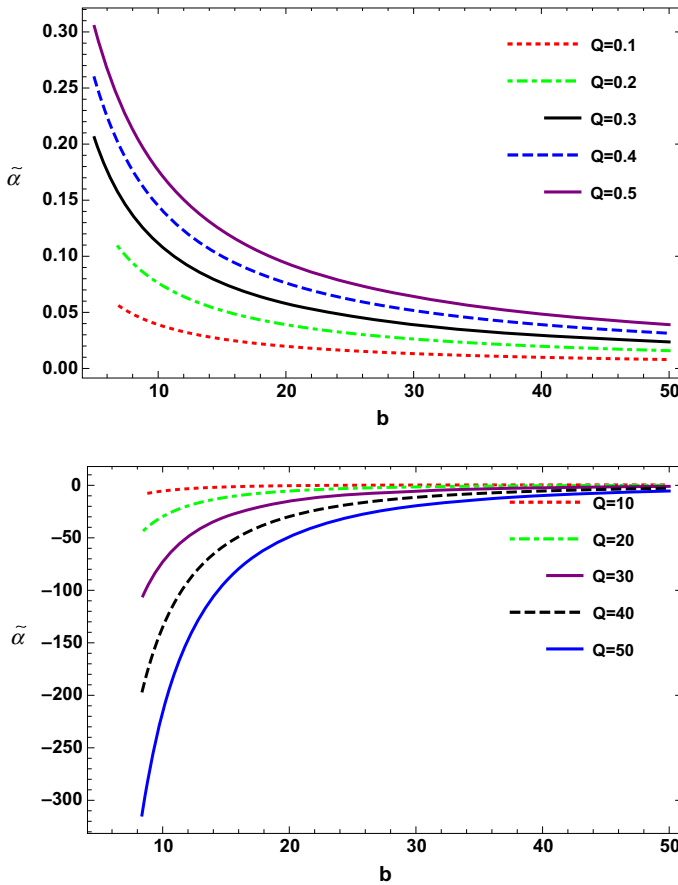$$F(r) = 1 + \frac{Q}{r} + \frac{Q^2\alpha}{8r(r+Q)^3}. \tag{16}$$

Fig. 1 $\tilde{\alpha}$ versus b

Now, the value of Gaussian curvature is found as:

$$\mathcal{K} \approx -2\frac{Q}{r^3} + \left(12\,r^{-4} - 4\frac{\alpha}{r^6}\right)Q^2 + \frac{\omega_e^2}{\omega_\infty^2}\left(\frac{-3Q}{2r^3} + \frac{8Q^2}{r^4} - \frac{3Q^2\alpha}{r^6}\right) + \mathcal{O}(Q^3, \alpha^2).$$

(17)

For this, we use GBT to compute the deflection angle in order to relate it with non-plasma. As follows, for calculating angle in the weak field area, the light beams become a straight line . Therefore, we use condition of $r = \frac{b}{\sin\phi}$ at zero order.

$$\tilde{\alpha} = -\lim_{R\to 0}\int_0^\pi \int_{\frac{b}{\sin\phi}}^R \mathcal{K}\mathrm{d}S$$

(18)

So, the deflection angle in the presence of plasma medium is defined as

$$\tilde{\alpha} = \frac{4Q}{b} - \frac{3Q^2\pi}{b^2} + \frac{3Q}{b}\frac{\omega_e^2}{\omega_\infty^2} - \frac{2Q^2}{b^2}\frac{\omega_e^2}{\omega_\infty^2} + \mathcal{O}(Q^3, b^3).$$

(19)

The above results tell us that photon rays are moving into medium of homogeneous plasma. We see that Eq. 19 reduced into Eq. 13 when plasma effect is removed.
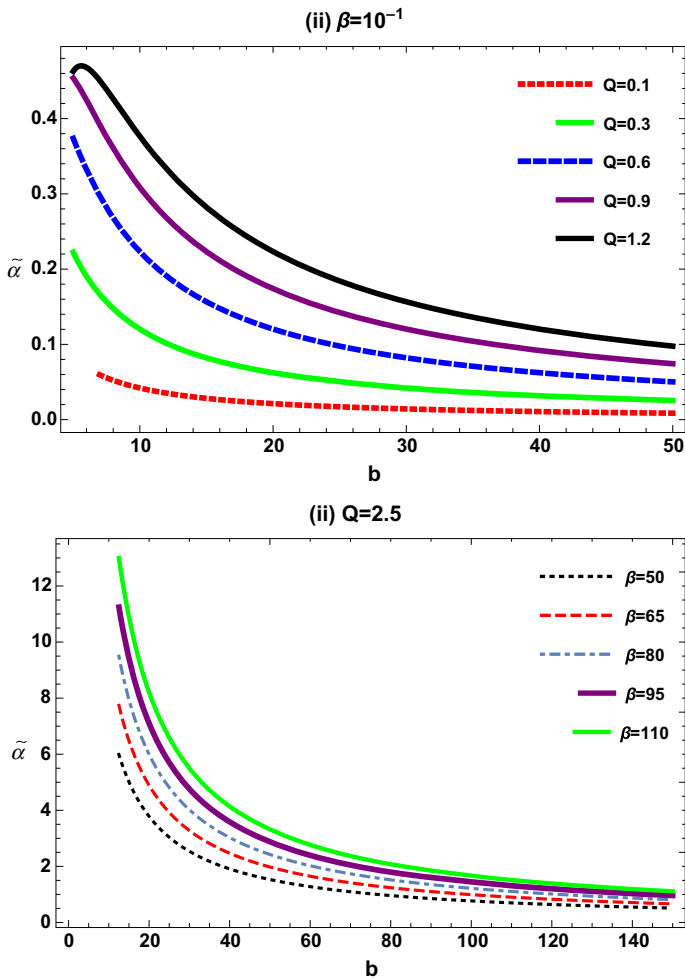
Fig. 2 $\tilde{\alpha}$ versus b

5 Graphical analysis for plasma medium

This section give us detailed graphical analysis of deflection angle $\tilde{\alpha}$ in the presence of plasma medium. We discussed the effect of black holes charge Q, impact parameter b and β on deflection angle for this, we take $\beta = \frac{\omega_e^2}{\omega_\infty^2} = 10^{-1}$.

- Figure 2, the left plot provides us behavior of deflection angle $\tilde{\alpha}$ with impact parameter b by changing value of Q and taking fixed value of β. We determine that for small values of Q, the deflection angle exponentially decreased and then approached to positive infinity, but as we choose high values of Q, the behavior of deflection angle is unstable. Thus, we summarized that as we choose small values for Q, the behavior of deflection is stable. The behaviors of deflection angle $\tilde{\alpha}$ with b by changing the values of β and for fixed Q have been analyzed in right plot. We examined that the deflection angle decreases gradually and moves toward positive infinity.

6 Conclusion

The present paper is about the investigation of deflection angle by stringy BH in plasma and non-plasma medium. Initially, we study the weak gravitational lensing by using GBT and derived the deflection angle of photon. Then, our proposed deflection angle is as follows,

$$\tilde{\alpha} = \frac{4Q}{b} - \frac{3Q^2\pi}{b^2} + \mathcal{O}(Q^3, b^3).$$

The above deflection angle is reduced to Schwarzschild-like deflection angle respect to first-order terms when $Q = M$. We have also discussed the graphical effect of different parameters on deflection angle by stringy BH. We have observed that, there is a direct relation between deflection angle and impact parameter while to obtain the stable behavior of deflection angle we only choose $0 < Q \le 1$ and negative α.

We have managed to expand the range of our research by comprising the influence of the plasma medium also on the angle of deflection given by Eq. (19) which is;

$$\tilde{\alpha} = \frac{4Q}{b} - \frac{3Q^2\pi}{b^2} + \frac{3Q}{b}\frac{\omega_e^2}{\omega_\infty^2} - \frac{2Q^2}{b^2}\frac{\omega_e^2}{\omega_\infty^2} + \mathcal{O}(Q^3, b^3).$$

If we neglected the $\frac{\omega_e}{\omega_\infty}$ term plasma effect can be removed. Additionally, we have demonstrated the graphical impact of deflection angle on stringy BH in a plasma medium.

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